Opinionated article

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Ultralow threshold blue quantum dot lasers: what's the true recipe for success?

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Abstract: The family of III-nitride materials has provided a platform for tremendous advances in efficient solid-state lighting sources such as light-emitting diodes and laser diodes. In particular, quantum dot (QD) lasers using the InGaN/GaN material system promise numerous benefits to enhance photonic performance in the blue wavelength regime. Nevertheless, issues of strained growth and difficulties in producing InGaN QDs with uniform composition and size pose daunting challenges in achieving an efficient blue laser. Through a review of two previous studies on InGaN/GaN QD microdisk lasers, we seek to provide a different perspective and approach in better understanding the potential of QD emitters. The lasers studied in this paper contain gain material where QDs are sparsely distributed, comprise a wide distribution of sizes, and are intermixed with "fragmented" quantum well (fQW) material. Despite these circumstances, the use of microdisk cavities, where a few distinct, high-quality modes overlap the gain region, not only produces ultralow lasing thresholds (~6.2 μJ/cm^2) but also allows us to analyze the dynamic competition between QDs and fQWs in determining the final lasing wavelength. These insights can facilitate "modal" optimization of QD lasing and ultimately help to broaden the use of III-nitride QDs in devices.

Keywords: blue semiconductor laser; InGaN/GaN; microdisk cavity; quantum dot laser.

1 Introduction

Microscale light sources have seen impressive advances in sophistication and applicability over the past few decades. The development of increasingly miniaturized light sources has helped realize new technologies across a variety of research fields such as quantum computing, photonic integrated circuits, displays, and biomedicine [1–7]. When research into light-emitting diodes (LEDs) and laser diodes (LDs) began, III–V compound semiconductors (i.e., GaAs: gallium arsenide, InP: indium phosphide, GaN: gallium nitride, etc.) emerged as promising candidate materials owing to their direct bandgaps and high carrier mobilities [8, 9]. Basic research of these devices started with longer wavelength-emitting source materials. In the 1960s, Ga(As$_{1-x}$P$_x$), GaAs, and InP diodes were shown to emit stimulated, coherent light in the red and infrared (IR) wavelength regimes [10–12]. As the sophistication of epitaxial growth techniques progressed, quantum heterostructures, beginning with quantum wells (QWs), were studied as enhanced gain materials for LDs. In the 1970s, researchers began demonstrating the use of gallium aluminum arsenide (GaAlAs) QWs in IR LDs [13]. To further improve carrier confinement and material gain, theoretical studies on quantum dots (QDs) began in the early 1980s, and the first GaAs QD laser was shown in 1994 [14–16].

While these discoveries paved the way for adoption of longer wavelength LEDs and LDs in commercial markets, research into high-power UV/blue light sources lagged. This was in part due to challenges associated with growing and doping high-quality, epitaxial layers of gallium nitride (GaN), one of the III–V semiconductors able to emit light in the UV/blue regime [17]. GaN became sought after because of inherent benefits over other semiconductors, including its large bandgap, a large exciton binding energy (allowing it to operate at room temperature more readily), and the ability to emit over the entire visible spectrum through the ternary alloys, Al$_y$Ga$_{1-y}$N and In$_x$Ga$_{1-x}$N [18]. However, the lack of lattice-matched growth substrates for GaN and other fabrication difficulties overshadowed these advantages of GaN and paused many research endeavors in GaN-based devices relative to semiconductors like GaAs. Stimulated emission in the UV/blue regime was demonstrated in the 1970s with optically pumped GaN single-crystal needles [19]; however, injection-based GaN LEDs and LDs would not see high-quality production for another

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20 years. It wasn't until the early 1990s that these devices took off with Nakamura et al.'s fabrication of GaN LEDs with GaN buffer layers on top of sapphire substrates [17, 20]. Shortly after, in 1996, using a similar fabrication technique, Nakamura et al. [21] also fabricated the first efficient GaN-based LDs with indium gallium nitride (InGaN) multiple QWs as the gain medium.

Since these developments, InGaN QDs embedded in GaN-based lasers have grown into an exciting area of research to improve gain characteristics and lasing thresholds further. Self-assembled InGaN QDs on GaN have been successfully fabricated through a variety of techniques, including metalorganic chemical vapor phase deposition, molecular beam epitaxy, and modified droplet epitaxy (MDE) [22–24]. However, the same challenges that plagued the growth of high-quality GaN also apply to the controlled formation of InGaN QDs and QWs as gain materials for lasers (as compared to other III–V devices where lattice-matched or nearly lattice-matched substrates are available). Lattice mismatch–induced strain between GaN and sapphire (the most commonly used underlying substrate) can produce threading dislocations which can degrade device performance [25]. Additionally, fabricating the "ideal" array of uniformly sized and spaced InGaN QDs is challenging, and MDE can result in patchy areas of QWs accompanying the QDs [23].

Despite these defects, InGaN QDs have been shown to perform effectively, especially when placed within high-quality optical microcavities. Our research has been at this critical juncture between the two major design specifications of an ultralow threshold, microscale semiconductor laser in the blue wavelength regime: (1) choosing an effective gain material and (2) designing a microscale optical cavity with only a few, high-quality modes. Through fabricating undercut GaN-based microdisk cavities, we have developed an effective "test bed" for exploring the dynamics of low-threshold lasing. Our optically pumped devices contain a unique heterogeneous gain material that includes InGaN QDs and "fragmented" QWs (fQWs), which are a by-product of the MDE method mentioned above. One would expect this nonuniform gain material to be a problem leading to poor device performance and high thresholds. However, through our experiments, we have found the opposite to be true. By first exploring the distinct lasing signature of InGaN QDs and then designing microring cavities, we have observed remarkable behavior, including the consistent dominance of InGaN QDs in the lasing process and modal engineering strategies to push lasing thresholds to still lower values. These insights can help improve understanding of the fundamental lasing dynamics of blue QD lasers, ultimately advancing a wide variety of applications.

2 Gain material

The first consideration in designing a microscale, low-threshold laser is the selection of the gain, or active, material. The gain material acts as an emitter, facilitating exciton recombination to generate photons and promote stimulated light emission [26]. The gain material also determines the laser's general wavelength regime, which can be tailored by using quantum heterostructures. Quantum heterostructures are composed of alternating materials with different bandgaps to produce a potential well with defined energy states where carriers can be captured and thereafter recombine radiatively. The two quantum heterostructures relevant to our research include InGaN QWs and QDs. QWs consist of slabs of InGaN sandwiched between two layers of GaN, leading to a potential energy trap that can confine electron-hole pairs. The slab-like QW structure confines carriers in one dimension, causing it to have a step-like density of states [15]. QDs also trap carriers through potential energy differences between the InGaN dot and the surrounding GaN. However, QDs are shaped like boxes and are much smaller than QWs. QDs confine carriers in three dimensions rather than one, which leads to a variety of advantages over the QW system. The increased confinement restrains carrier diffusion, compared to the situation for QWs, localizing the electron-hole pairs and making QDs less susceptible to defects such as threading dislocations [27]. InGaN QDs also have a higher probability than QWs to produce radiative recombination between carriers because they are less affected by the material's built-in electric field (owing to the polar *c*-axis of the InGaN wurtzite crystal structure), which separates electron and hole wave functions [28]. Finally, unlike QWs, QDs have a delta-like density of states, which in theory leads to higher material gain with a narrowed spectrum [15]. In addition to using QWs and QDs in isolation, there has been research in combining them to capitalize on each of their advantages. This combination can be known as dots-in-well (DWELLs), or quantum well-dots (QWDs), where indium-rich QDs are placed within a QW [29]. QWs have a much higher probability of carrier capture than QDs owing to their larger size, which may allow them to capture electron-hole pairs and then funnel them to QDs, ultimately enhancing the inherent benefits of QDs mentioned above.

In theory, a perfect QD array should provide the best gain characteristics and spontaneous recombination rates and, therefore, the lowest lasing thresholds [15]. This should be especially true when the array is placed in a cavity with high-quality modes; the interaction between the QDs and the cavity modes produce a Purcell

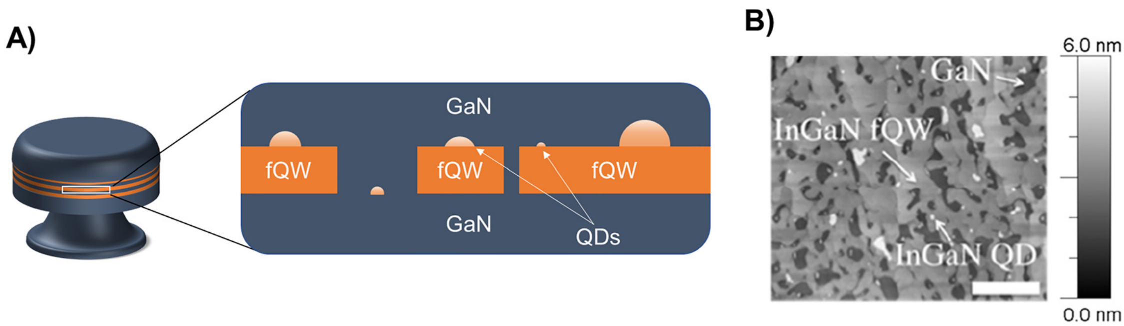

Figure 1: Our microdisk platform.
(A) Schematic of a 1-μm-diameter microdisk laser with three layers of InGaN active material. Inset shows one layer of InGaN fQWs and QDs, showing how QDs form on the fQWs and underlying GaN. (B) An atomic force microscopy (AFM) image of active material, specifying GaN, InGaN fQW, and InGaN QD regions. Taken from the study by Woolf et al. [32]. Scale bar is 500 nm in length, and vertical color scale goes from 0 to 6 nm. QD, quantum dot; fQW, fragmented quantum well; InGaN, indium gallium nitride.

enhancement of the QD emission [30]. However, decreased thresholds in InGaN QD lasers have not been seen experimentally, and InGaN QWs we have studied in undercut microdisk cavities consistently exhibit lower lasing thresholds than samples containing QDs [31]. One reason for this is that producing a "perfect" InGaN QD array with consistent QD size, consistent levels of In, and a sufficiently high density of QDs is quite challenging. While these difficulties are not unique to InGaN QDs (vs. other types of QDs), they can be exacerbated in the InGaN system owing to the material's inherent growth hurdles described above. Additionally, our QD samples are always accompanied with layers of fQWs. Thus, these InGaN QDs are in different potential environments: either directly on top of fQWs or on the underlying GaN. This effect is shown in Figure 1.

The InGaN QDs shown in Figure 1 were provided by Oliver et al. [23] at the University of Cambridge and fabricated via a MDE method. From the results displayed in Figure 1B, we see that owing to the challenges in growth mentioned above, QD formation seems to be randomly distributed over the GaN substrate and fQWs, with a variety of sizes. Additionally, the density of QDs and their individual sizes are quite small compared to the surrounding fQWs. In a given microdisk laser with a diameter of 1 μm and three layers of QDs with an density of 1×10^{10} cm^{-2}, this results in ~240 QDs per laser, with a very small surface area coverage compared to the fQWs. This sparse areal coverage dramatically lowers the probability of QDs to capture excitons compared to the larger fQWs. Therefore, one would expect the fQWs to have a greater contribution to the gain within the active region and to lasing. In fact, because of the only three or so narrow high-quality microcavity modes that overlap the gain region, we do see this when optically pumping our devices at low powers. At input power levels far below the lasing threshold, the large capture cross section of the fQWs results in the gain material predominantly emitting photons into a mode near the material's general background emission spectrum (~460 nm). However, as we increase our pumping power, the dominant emission in the cavity shifts to a shorter wavelength mode that is blue shifted from the center of the general background emission (~430 nm). In fact, this shorter wavelength mode corresponds to the emission wavelength of InGaN QD excitons [32, 33]. The device then ultimately lases at this wavelength associated with the QDs. This distinct signature of QD lasing was shown in a previous work by Woolf et al. [32], and a conceptual representation of this is shown in Figure 2.

Overall, the abrupt shift from the fQW mode to the QD mode with increasing input power underscores the powerful role played by microcavities with only a few, high-quality modes that overlap the gain area. Once

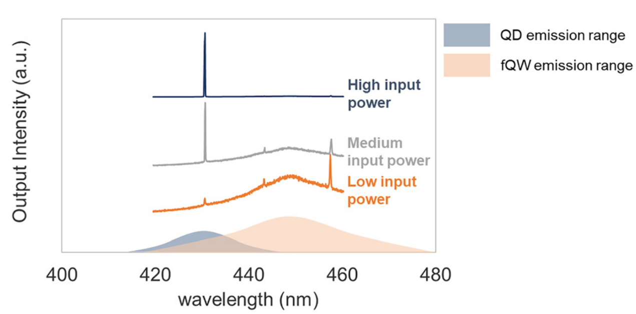

Figure 2: Illustrative example of the dominant mode shifting in a microdisk from the fQW regime to the QD regime with increasing input power. "Low input power" is far below the lasing threshold. "Medium input power" is slightly higher but still below the lasing threshold. "High power" is above the lasing threshold. Modes are indicated by the sharp peaks decorating the background emission. fQW, fragmented quantum well; QD, quantum dot.

enough photons are pumped into the cavity for the QDs to capture, the inherent advantage of having QDs with higher confinement and spontaneous recombination rates (than QWs) takes over. The Purcell effect further increases the QDs' radiative recombination rate through emitter-mode interactions, and the QDs' emission ultimately dominates the luminescence spectrum to achieve lasing.

3 Modal engineering

The research described in the previous section laid the foundation for us to look deeper into the second key component of a microdisk laser: the cavity modes that interact with our emitting material. In a disk-shaped optical cavity, the primary optical modes are whispering gallery modes (WGMs), which are produced by the total internal reflection of light traveling around the circumference of the disk. First-order WGMs are located at the periphery of the disk, while higher order radial modes encroach toward the center. Through finite-difference time-domain (FDTD) simulations, we confirmed that the first-order WGMs exhibit higher quality factors and are therefore better at capturing and storing photons in the microdisk than the higher order modes. Pairing these insights with the knowledge of our heterogenous gain material's unique lasing behavior discussed above, we then studied alterations to our microdisk geometry to see if we could push lasing thresholds lower.

For effective coupling between emitted light and the WGMs, the WGMs should overlap with the gain material spectrally and spatially. As stated in the previous section, our microdisk lasers containing QDs consistently lased via an optical mode with a wavelength of ~430 nm (or centered on the QD emission spectrum). This mode in a 1-micron-diameter disk, as determined through FDTD simulations, is a high-quality first-order mode near the periphery of the disk. Our simulations also reveal the presence of a lossy, higher order mode near the center of the fQW background emission [34]. This motivated us to explore cutting out the center of our microdisks to create microrings. This idea served two purposes. First, since our high-quality modes in the disks occur near the periphery, we could avoid injecting excess energy into the center of the disk where lasing modes do not reside, ultimately helping us lower the threshold. Second, since the low-quality, higher order modes occur closer to the center, by creating a ring, we could selectively remove photons with energies resonant with the lossy higher order modes while preserving photons that overlap with the WGMs near the periphery. This phenomenon is shown through our FDTD simulations in Figure 3, where rings with 200- and 500-nm inner diameters show the preservation of a high-Q WGM and degradation of a higher order mode corresponding to fQW photons.

Figure 3 presents the highest-Q WGM in the rings that occurs at a wavelength of 433 nm and a higher order 449-nm mode that overlaps the fQW spectral region. From the E-field profiles for both geometries, it is clear that the first-order 433-nm WGM appearing near the periphery of the disk is not significantly affected in either of the geometries, and the theoretical Q-factor remains high (~300,000 for each). For the higher order 449-nm mode, the inner circle of the 500-nm ring begins to overlap with the mode itself, which degrades that mode much more drastically than the first-order WGMs, allowing for the selective removal of nonlasing photons.

The idea of removing, or "leaking away" photons from a laser, and yet, still achieving lower threshold lasing, might at first thought seem to be logically inconsistent. In general, it is not possible to delineate the gain region into a "desired" gain material (QDs) and "less desirable" gain

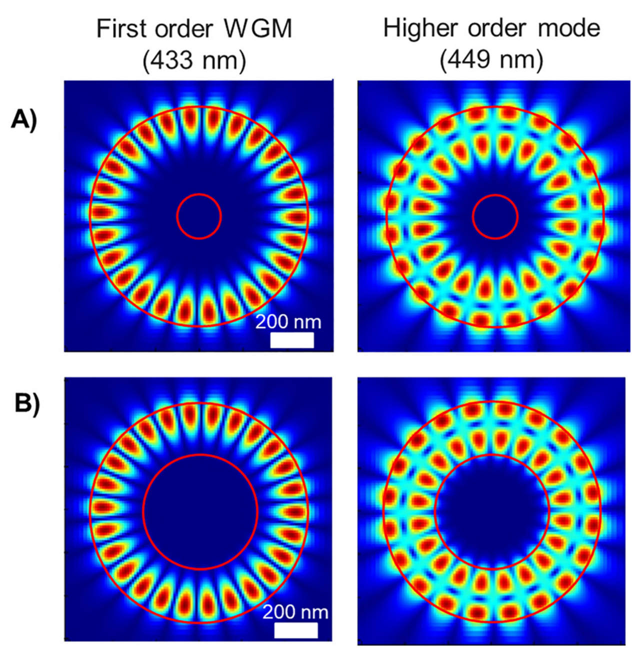

Figure 3: FDTD simulations of microrings.
(A) A microring with a 200-nm inner diameter and 1-μm outer diameter. The concentric red circles superimposed on the E-field profiles represent the inner and outer diameters of the structure. Here, we show a first-order WGM at a wavelength of 433 nm and a higher order mode at a wavelength of 449 nm. (B) A microring with a 500-nm inner diameter and 1-μm outer diameter. Once again, the red circles mark the geometry of the microring, and we present the same modes at 433 and 449 nm shown in part (A). FDTD, finite-difference time-domain; WGM, whispering gallery mode.

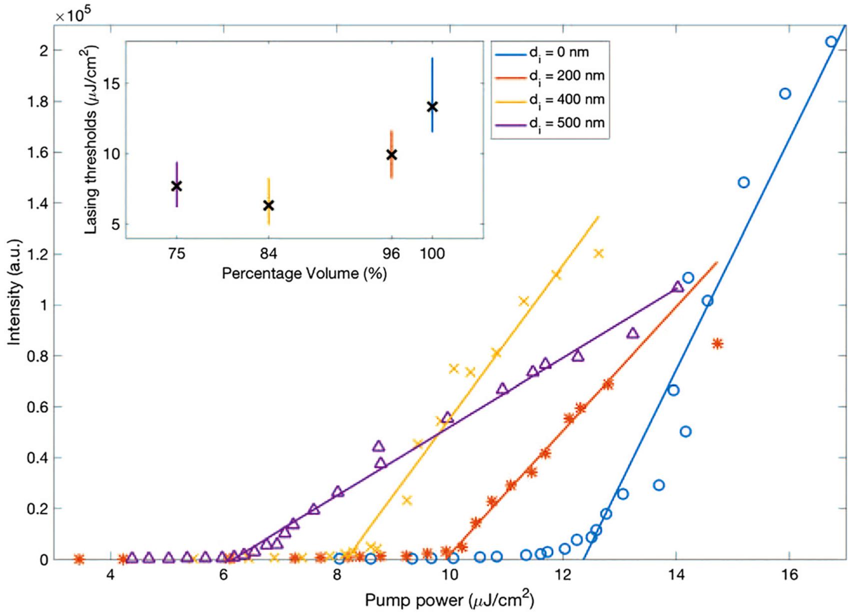

Figure 4: L–O curves for microrings with different inner diameter sizes (all had the same outer diameter length of 1 micron). Taken from the study by Wang et al. [35]. The inset shows average lasing thresholds (denoted by *x*'s) for the different-sized rings, each averaged over eight samples. L–O, Light-in–light-out.

materials (fQWs). Yet, as shown in Figure 2, there are fairly distinct regions separating the shorter wavelength QD emission from the broader, longer wavelength fQW emission. Even with such a heterogeneous gain spectrum, how is it possible to controllably modulate the cavity interaction with the two gain regions, selectively removing fQW photons while ensuring high cavity interaction with QD photons? However, part of the beauty and power of working with microcavity structures is being able to carry out this "modal engineering," as was demonstrated experimentally by Wang et al. [35] and illustrated in Figure 4.

Light-in–light-out (L–O) curves for microring devices with different geometries (different inner diameters or central areas removed) showed dramatically lower lasing thresholds (as low as 6.2 μJ/cm^2) than microdisks having the same value for the outer radius [35]. The selected range of inner radii (i.e., 0–500 nm) allowed us to explore the effect of removing the inner area while at the same time ensuring that the largest inner radius (500 nm) would not degrade the high-Q WGMs near the periphery. Moreover, Figure 4 also shows a decrease in the slope efficiency of the L–O curves with an increasing inner diameter for the rings. The lower slope efficiency reflects the greater loss of photons, but the majority of those photons are emitted from the fQWs and do not contribute to lasing, as suggested by our simulations in Figure 3.

As was true for the microdisks, as we pump these microrings at successively higher optical powers, the ~460-nm mode associated with the fQWs dominates at low powers, and as this input power increases, the ~430-nm mode associated with the QDs eventually wins out and ultimately achieves lasing for the device. Once again, even with fewer QDs in the microrings that could be contributing to the lasing modes, QDs still dominate the lasing process, confirming that the QDs' inherent advantages and the cavity's high-quality modes are unaffected in the ring geometry. And these rings produce even lower thresholds, highlighting the advantage of having a heterogeneous gain material emitting in distinct wavelength regimes to selectively remove unwanted photons.

4 Conclusion

We have studied lasing dynamics in blue QD lasers through a unique platform combining undercut GaN microdisk cavities with a heterogeneous gain material composed of InGaN QDs and fQWs. The spectral precision of the WGMs of the microdisk allows us to track the process of electron-hole capture in the gain medium at differing powers. The emitter-cavity interaction depends on both the spatial profiles of the high-quality modes overlapping with the gain material, as well as the resonance in frequencies. Although WGMs overlap the fQW spectral region as well as the QD spectral region, lasing ultimately occurs at the QD wavelengths. The initial advantages of fQWs with a broader capture area, and a larger spectral range, interacting with the WGMs ultimately lose out to the shorter spontaneous emission lifetimes of the QD emitters, more strongly coupling to the WGMs. Thus, QD emission consistently dominates the lasing process in our devices despite a multitude of factors working against it, including low QD carrier capture probabilities, threading dislocations, and inhomogenous broadening, owing to variations in QD size. Additionally, by cutting out the center of our microdisks to create microrings, we were able to remove photons (from the fQWs) that do not contribute to the lasing process, resulting in lower thresholds. This was possible due to our heterogeneous gain material, emitting photons of different wavelengths into engineerable higher and lower quality optical modes.

For further improvement of our devices, several areas can be explored. For example, the contributions to lasing from the different types of QD configurations (i.e., QDs sitting directly on GaN vs. sitting on top of an fQW) could be modeled to enhance our understanding. Do the fQWs help facilitate carrier capture and direct carriers to the QDs (like a DWELL or QWD), therefore, being more advantageous than a QD by itself embedded within the GaN cavity? Advanced cavity geometry studies beyond microdisks could also provide a wealth of knowledge to further capitalize on our gain material's unique emission behavior. Specifically, inverse design of optical cavities has been a burgeoning area of research [36], and we could adapt this design strategy to account for our InGaN fQW/QDs' distinct material properties. While QD placement within cavities is particularly difficult, premapping of InGaN QDs in bulk material and formation of smaller volume cavities around them could be explored to potentially achieve even lower thresholds. Finally, comparisons between optical pumping and electrical injection of our devices could be studied to determine if the trends we have observed with regard to lasing persist.

Ultimately, both studies highlighted in this paper have provided remarkable insight into low-threshold lasing in blue microcavity lasers and the superior qualities of InGaN QDs coupled with high-quality modes. Despite the initial delay and challenges in development of miniaturized commercial blue lasers compared to longer wavelength regimes, we believe our insights have uncovered hidden advantages of particular kinds of InGaN QDs that could be used to improve device performance while the optimization of QD growth itself advances. By studying this dynamic interplay between these two ingredients, namely, the gain material and optical cavity design, it is our hope that our insights can help further inform the quest to find the best recipe for an efficient blue QD laser.

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