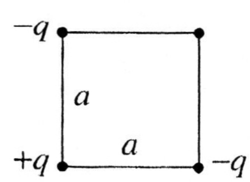
USING VECTOR CALCULUS TO SOLVE PROBLEMS IN ELECTRICITY AND MAGNETISM

Summer 2020
Zoom Lecture: F: 2:00-4:00 p.m.
National Science Foundation (NSF) Center for Integrated Quantum Materials (CIQM), DMR -1231319
Dr. Steven L. Richardson (srichards22@comcast.net)
Professor Emeritus of Electrical Engineering, Department of Electrical and Computer Engineering, Howard University, Washington, DC
and
Faculty Associate in Applied Physics, John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA

PROBLEM SET X
(due September 1 2020)

Problem 1

Three charges are situated at the corners of a square of side a as shown in the figure below. How much work does it take to bring another charge $+q$ from far away and place it in the fourth corner?



Problem 2

Refer to the system previously discussed in Problem 1. How much work does it take to assemble the whole configuration of four charges?

Problem 3

Consider an infinite one-dimensional chain of point charges q and $-q$ with alternating signs strung out along the x-axis each with a distance a from its nearest neighbor. Compute the work per particle to assemble this system. As a hint consider the Maclaurin series expansion of

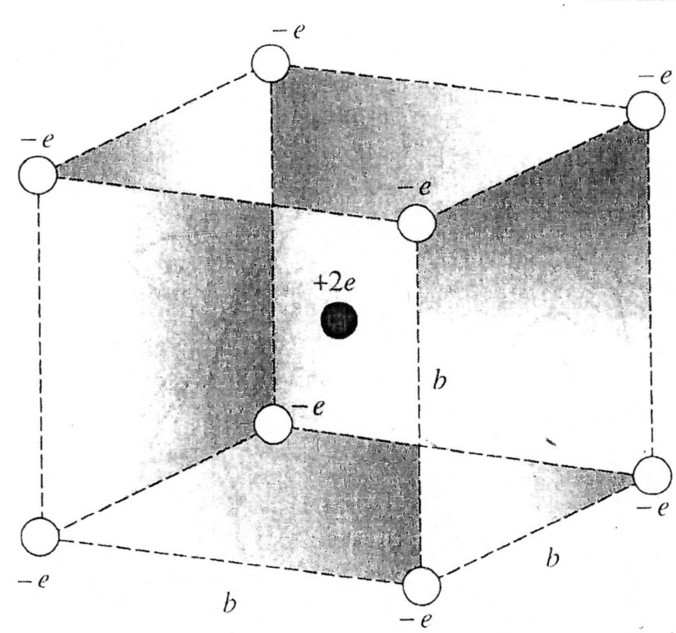
$$\ln(1+x)$$

Problem 4

Two positive charges point charges q_a and q_b of masses m_a and m_b respectively are at rest held together by a massless string of length a. If the string is cut, the two particles fly off in opposite directions. How fast is each one going when they are far apart?

Problem 5

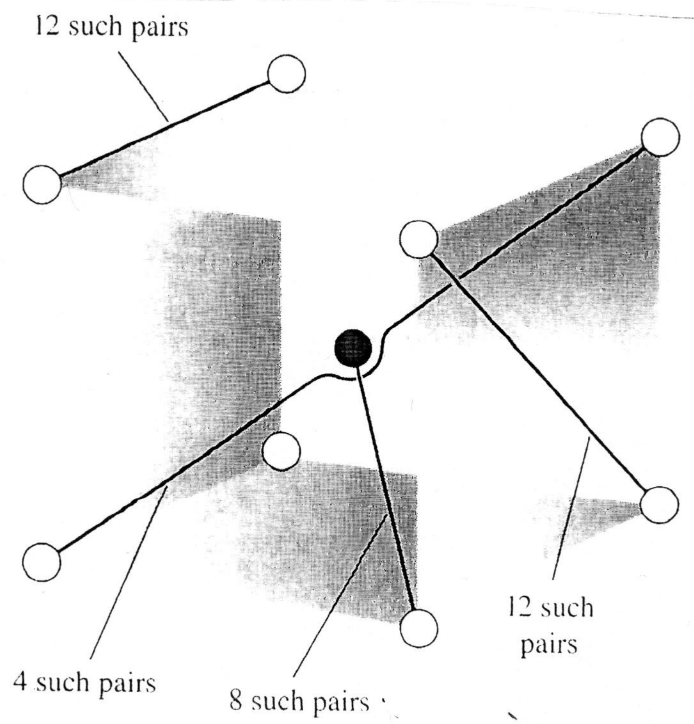
Show that the electrostatic potential energy U of an arrangement of eight separate negative charges $-q$ on the corners of a cube of side b with a positive charge $+2q$ located at the center of the cube.



is given by

$$U = \frac{1}{4\pi\epsilon_o}\left[\frac{-16\,q^2}{\frac{\sqrt{3}}{2}\,b} + \frac{12\,q^2}{b} + \frac{12\,q^2}{\sqrt{2}\,b} + \frac{4\,q^2}{\sqrt{3}\,b}\right] = \frac{q^2}{4\,b\,\pi\,\epsilon_o}\,4.31947$$

You can use as a hint the figure below where you should recall that we only have to worry about calculating pairs of interactions among charges when evaluating the electrostatic potential energy of a system.

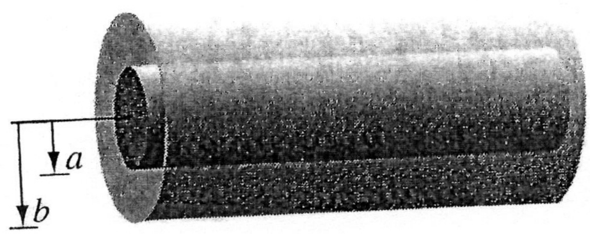


Problem 6

Find the capacitance of two coaxial spherical metal shells with radii r_a and r_b where $r_a < r_b$. Note that the inner sphere has a charge $+q$ while the outer sphere has a charge $-q$.

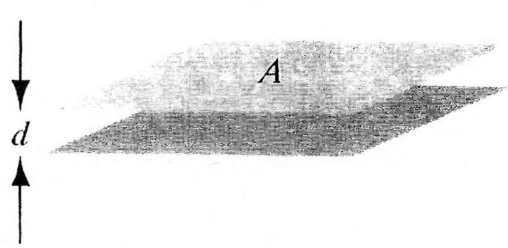
Problem 7

Find the capacitance per unit length of two coaxial metal cylindrical tubes of radii r_a and r_b where $r_a < r_b$ as shown in the figure below. Note that the inner cylinder has a surface charge density $+\sigma$ while the outer cylinder has a surface charge density $-\sigma$.



Problem 8

Consider the parallel-plate capacitor shown in the figure below



If you wish to "charge up" the capacitor, you have to remove electrons from the positive plate and move them to the negative plate. In doing so, you fight against the electric field which is pulling them back towards the positive plate and pushing them away from the negative one. How much work is then done to charge the capacitor up to some final amount Q? We can calculate this by supposing that at some intermediate stage the charge on the positive plate is q. This would mean that the electrostatic potential difference between the two plates of the capacitor is $\frac{q}{C}$. The work required to move the next piece of charge dq is therefore

$$dW = \frac{q}{C}\,dq$$

Show that the total work required to go from $q = 0$ to $q = Q$ is

$$W = \frac{1}{2}CV^2$$

where V is the final electrostatic potential of the capacitor.