USING VECTOR CALCULUS TO SOLVE PROBLEMS IN ELECTRICITY AND MAGNETISM

Summer 2020
Zoom Lecture: F: 2:00-4:00 p.m.
National Science Foundation (NSF) Center for Integrated Quantum Materials (CIQM), DMR -1231319
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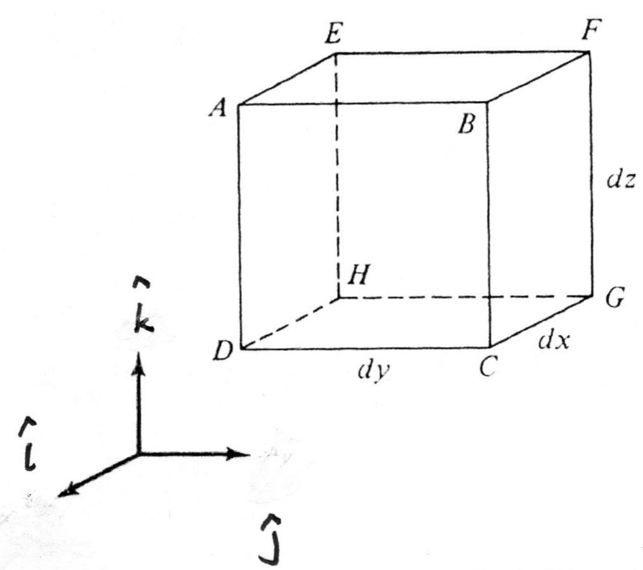
PROBLEM SET III
(due Friday, July 10, 2020)

Problem 1

Here is a picture of the differential volume element in Cartesian coordinates

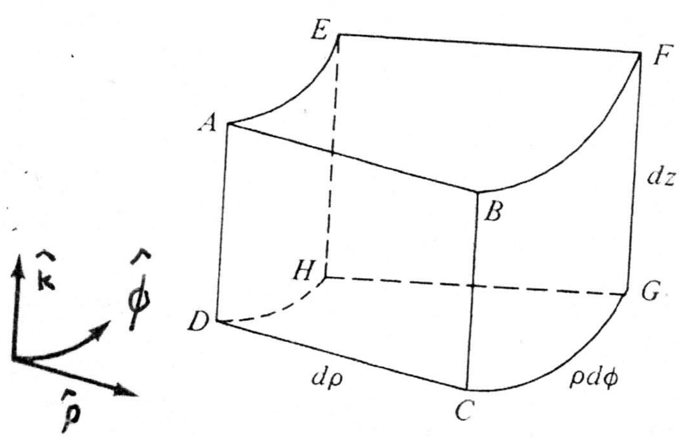


Using this figure match the items in the left column with those in the right column.

(a) $d\vec{r}$ *from* A *to* B	(i) $dy\,dz\,\hat{\imath}$
(b) $d\vec{r}$ *from* A *to* D	(ii) $-\,dx\,dz\,\hat{\jmath}$
(c) $d\vec{r}$ *from* A *to* E	(iii) $dx\,dy\,\hat{k}$
(d) $d\vec{S}$ *for face* $ABCD$	(iv) $-\,dx\,dy\,\hat{k}$
(e) $d\vec{S}$ *for face* $AEHD$	(v) $-\,dx\,\hat{\imath}$
(f) $d\vec{S}$ *for face* $DCGH$	(vi) $dy\,\hat{\jmath}$
(g) $d\vec{S}$ *for face* $ABFE$	(vii) $-\,dz\,\hat{k}$

Problem 2

Here is a picture of the differential volume element in cylindrical polar coordinates. You should realize that although it does not look like a box as in the previous problem it becomes one for the small differentials used. Please appreciate the beauty of the differential in calculus!

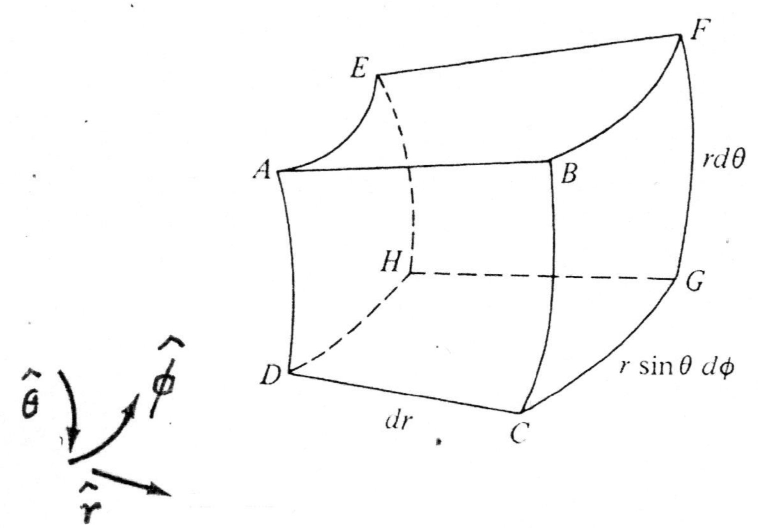


Using this figure match the items in the left column with those in the right column.

(a) $d\vec{r}$ *from* E *to* A	(i) $-\rho\, d\phi\, dz\, \hat{\rho}$
(b) $d\vec{r}$ *from* B *to* A	(ii) $-d\rho\, dz\, \hat{\phi}$
(c) $d\vec{r}$ *from* D *to* A	(iii) $-\rho\, d\rho\, d\phi\, \hat{k}$
(d) $d\vec{S}$ *for face* $ABCD$	(iv) $\rho\, d\rho\, d\phi\, \hat{k}$
(e) $d\vec{S}$ *for face* $AEHD$	(v) $-d\rho\, \hat{\rho}$
(f) $d\vec{S}$ *for face* $ABFE$	(vi) $-\rho\, d\phi\, \hat{\phi}$
(g) $d\vec{S}$ *for face* $DCGH$	(vii) $dz\, \hat{k}$

Problem 3

Here is a picture of the differential volume element in spherical polar coordinates. You should realize that although it does not look like a box as in the Problem 1 it becomes one for the small differentials used. Please appreciate the beauty of the differential in calculus!



Using this figure match the items in the left column with those in the right column.

(a) $d\vec{r}$ *from* A *to* D	(i) $-r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$
(b) $d\vec{r}$ *from* E *to* A	(ii) $-r \sin\theta \, dr \, d\phi \, \hat{\theta}$
(c) $d\vec{r}$ *from* A *to* B	(iii) $r \, dr \, d\theta \, \hat{\phi}$
(d) $d\vec{S}$ *for face* $EFGH$	(iv) $dr \, \hat{r}$
(e) $d\vec{S}$ *for face* $AEHD$	(v) $r \, d\theta \, \hat{\theta}$
(f) $d\vec{S}$ *for face* $ABFE$	(vi) $-r \sin\theta \, d\phi \, \hat{\phi}$

Problem 4

Using the figures in Problems 1, 2, and 3, prove to yourself that the following expressions are true. It is very important that you see this!

(4-1) The displacement vectors or differential differentials in Cartesian, cylindrical polar, and spherical polar coordinates are

$$d\vec{r} = dx \, \hat{\imath} + dy \, \hat{\jmath} + dz \, \hat{k}$$

$$d\vec{r} = d\rho \, \hat{\rho} + \rho \, d\phi \, \hat{\phi} + dz \, \hat{k}$$

$$d\vec{r} = dr \, \hat{r} + r \, d\theta \, \hat{\theta} + r \sin\theta \, d\phi \, \hat{\phi}$$

(4-2) The differential surface elements in Cartesian, cylindrical polar, and spherical polar coordinates are

$$d\vec{S} = dy\,dz\,\hat{\imath} + dx\,dz\,\hat{\jmath} + dx\,dy\,\hat{k}$$

$$d\vec{S} = \rho\,d\phi\,dz\,\hat{\rho} + d\rho\,dz\,\hat{\phi} + \rho\,d\rho\,d\phi\,\hat{k}$$

$$d\vec{S} = r^2 \sin\theta\,d\theta\,d\phi\,\hat{r} + r\,\sin\theta\,d\phi\,dr\,\hat{\theta} + r\,dr\,d\theta\,\hat{\phi}$$

(4-3) The differential volume elements in Cartesian, cylindrical polar, and spherical polar coordinates are

$$dV = dx\,dy\,dz$$

$$dV = \rho\,d\rho\,d\theta\,dz$$

$$dV = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

Problem 5

Evaluate the following surface integral

$$\oint_S \vec{A} \cdot d\vec{S}$$

where

$$\vec{A} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$$

over the unit cube described by

$$0 \leq x \leq 1$$

$$0 \leq y \leq 1$$

$$0 \leq z \leq 1$$

Problem 6

Determine the flux of $\vec{G}$

$$\oint_S \vec{G} \cdot d\vec{S}$$

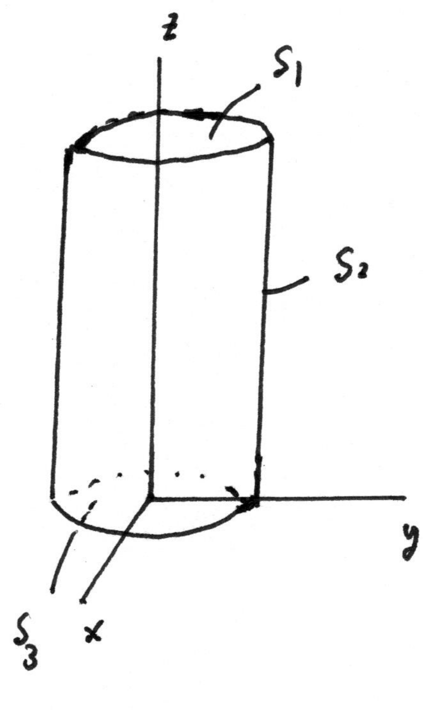
where

$$\vec{G}(r) = 10e^{-2z}(\rho\,\hat{\rho}_z + \hat{k})$$

out of the entire surface of the cylinder

$$\rho = 1$$

$$0 \leq z \leq 1$$



$S = S_1 + S_2 + S_3$

Problem 7

Determine the flux of $\vec{D}$

$$\oint_S \vec{D} \cdot d\vec{S}$$

where

$$\vec{D} = \rho^2 \cos^2 \phi \, \hat{\rho} + z \, \sin \phi \, \hat{\phi}$$

over the closed surface of the cylinder

$$\rho = 4$$

$$0 \leq z \leq 1$$

Problem 8

If

$$g(\theta, \phi) = r^2$$

evaluate

$$\int_V g(\theta, \phi) \, dV$$

over the hemisphere of radius 1 centered at the origin where

$$\rho = 4$$

$$z \geq 0$$

$$g(\theta, \phi) = r^2$$

Problem 9

Evaluate the line integral

$$\oint_S \vec{A} \cdot d\vec{r}$$

where

$$\vec{A} = \rho \cos\phi \, \hat{\rho} + z \, \sin\phi \, \hat{k}$$

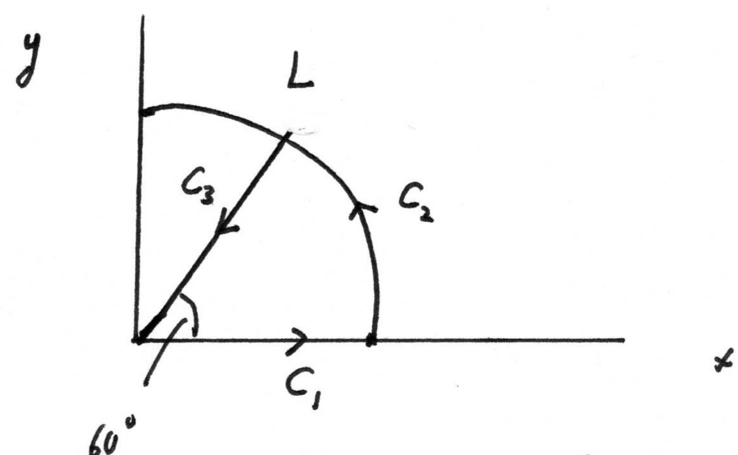
and C is the edge L of the wedge defined by

$$0 \leq \rho \leq 2$$

$$0 \leq \phi \leq 60^o$$

$$z = 0$$

as shown in the figure below



$L = C = C_1 + C_2 + C_3$

Problem 10

1. Error in Problem Set I (Solutions): On Page 3 you should have

$$\| \vec{u} \|^2 = \| \vec{v} \|^2$$

2. In the taped version of Lecture 1, I misspelled "Cartesian".