

Microwave boson sampling

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The first post-classical computation will most probably be performed not on a universal quantum computer, but rather on a dedicated quantum hardware. A strong candidate for achieving this is represented by the task of sampling from the output distribution of linear quantum optical networks. This problem, known as boson sampling, has recently been shown to be intractable for any classical computer, but it is naturally carried out by running the corresponding experiment. However, only small scale realizations of boson sampling experiments have been demonstrated to date. Their main limitation is related to the non-deterministic state preparation and inefficient measurement step. Here, we propose an alternative setup to implement boson sampling that is based on microwave photons and not on optical photons. The certified scalability of superconducting devices indicates that this direction is promising for a large-scale implementation of boson sampling and allows for more flexible features like arbitrary state preparation and efficient photon-number measurements.

INTRODUCTION

In the context of linear optics quantum computation, the fundamental work of Knill, Laflamme and Milburn [1] showed that a universal set of gates is implementable when deterministic single photon sources, efficient detectors and fast electronic feed-forward are exploited. Achieving any of these three ingredients constitutes an impressive technological challenge by itself, but the question is whether all these components are necessary to realize a form of computation superior to the one of classical computers. The answer, as explicitly put forward by Aaronson and Arkhipov [2], is no. In particular, no feed-forward loops are necessary and also the non-deterministic nature of state-of-the-art single photon sources can be partially tolerated [3, 4].

The task that Aaronson and Arkhipov proposed and showed to be, modulo a couple of reasonable conjectures, intractable for classical computers [2], is the simulation of a linear optical quantum network in which the input state of each mode corresponds to either a single photon or the vacuum state. The problem is to sample from the photon number distribution measured at the output of each mode. For this reason, it has been referred to as boson sampling. Such hardness proof is remarkably important since it shows that intermediate quantum setups can challenge the extended Church-Turing (ECT) thesis by suggesting a physical implementation that computes more efficiently than a non-deterministic Turing machine. In practice, the ECT thesis is not directly refutable since it refers to an asymptotically large scale implementation of a physical device, but the clear indication of a scalable setup and the neat experimental demonstration of such

computation in medium-size devices would constitute a serious indication to reconsider the ECT thesis.

The emphasis of the previous argument points to the scalability issue. In fact, the original boson sampling setup works with optical photons that are difficult to generate as single photons in a deterministic way and that, given the state-of-the-art, cannot be detected with almost unit efficiency. Subsequent proposals have suggested the use of different initial states, like two-mode squeezed states [4], photon added/subtracted coherent states [5], or vacuum squeezed states [6]. These modifications only partially solve the bottlenecks of non-deterministic state preparation and detection efficiency making the actual implementation of boson sampling exponentially demanding in the number of photons [7, 8]. A different approach to overcome such difficulties is the use of alternative experimental setups. Phonons are bosonic particles and, under corresponding Hamiltonians, behave in the same way as photons. Shen, Zhang and Duan proposed to use trapped ions and their collective vibrations to implement boson sampling [9]. Unfortunately, the required interactions are not the natural ones for the setup considered, so frequent and localized laser pulses are necessary to constantly alter the dynamics with active control techniques. This overhead limits the applicability of this construction to a small number of vibrational modes of the trapped ions.

In this article, we propose to realize boson sampling with photons outside the optical regime, in particular we show how microwave photons are ideal for a scalable implementation that takes into account all three fundamental steps of the problem: I) deterministic state preparation, II) direct implementation of the appropriate dynamics, and III) highly efficient measurements. In our proposal, we substitute the open-end optical waveguides with identical superconducting resonators, one for each mode, and couple them through a superconducting ring coupler implementing a tunable beam splitter Hamilto-

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nian. Phase shifters are naturally implemented by tuning the resonator frequency in an independent way with the aid of an adjacent superconducting qubit. In this setup, state preparation is efficiently carried out by loading the corresponding state of the qubit into each resonator using the Jaynes-Cummings interaction in circuit quantum electrodynamics (circuit QED) [10, 11]. The introduction of additional low-quality-factor (low-Q) resonators allows the system readout through a quantum non-demolition measurement. Note that in this proposal, the spatial degree of freedom of the waveguides is replaced by a series of controlled steps in time evolution, as shown in Figure 2. All the above operations can be performed deterministically and with high fidelity on state-of-the-art superconducting devices with little design modifications with respect to current setups [12]. As demonstrated independently in [13] and in [14, 15], these missing ingredients are relatively easy to integrate in superconducting architectures. This guarantees the scalability of our proposal and suggest superconducting platforms as a major physical candidate to the realization of large scale boson sampling experiments.

Finally, the advantages of the proposed implementation do not only help us to address computational complexity questions alone, even if of primary importance, they also have a second, more practical relevance. Recently, a modified version of the original boson sampling apparatus has been shown to be an essential component of the quantum simulation of molecular vibronic spectra [16]. The additional operations required to achieve such simulation are, essentially, the application of displacement and squeezing operations. These operations are readily carried out using superconductors by means of the manipulation of the initial state of the photons [17]. Our proposal could then pave the way to the first boson sampling experiment with direct practical implications.

BOSON SAMPLING HAMILTONIAN

The dynamics of passive linear optical systems is determined by the sequence of beam splitter and phase shifting elements that constitute the photonic network. Here, we show how their action can be described in terms of the sequential application of specific Hamiltonians.

In its original formulation, boson sampling refers to the situation in which N single photons are injected in a M -modes photonic network characterized by the unitary matrix U . Introducing the Fock number basis, *i.e.* the basis composed by states $\{|n_1, n_2, \dots, n_M\rangle\}$ having a precise number of photons n_j in each mode $j = 1, 2, \dots, M$, we can write the input and output state as

$$|\psi_{\text{in}}\rangle = |1_1, \dots, 1_N, 0_{N+1}, \dots, 0_M\rangle, \quad (1)$$

$$|\psi_{\text{out}}\rangle = \hat{R}_U |\psi_{\text{in}}\rangle, \quad (2)$$

where the transformation \hat{R}_U is defined through its action on the bosonic creation operators by $\hat{R}_U a_i^\dagger \hat{R}_U^\dagger =$

$\sum_j U_{ij} a_j^\dagger$. Aaronson and Arkhipov showed that sampling from the photon-number output distribution $P(n_1, n_2, \dots, n_M) = |\langle n_1, n_2, \dots, n_M | \hat{R}_U |\psi_{\text{in}}\rangle|^2$ is a computationally hard task, provided that the number of modes $M \geq N^2$ and that the unitary U is chosen randomly according to the Haar measure [2].

Since any linear optical network can be constructed with phase shifters (ps) and beam splitters (bs) alone, \hat{R}_U can also be decomposed as the sequential product of the corresponding unitary operations acting, respectively, only on one or two modes. The constructive proof that any $M \times M$ unitary matrix U can be associated with a photonic network composed by $K = \mathcal{O}(M^2)$ optical elements [18] provides the factorization $\hat{R}_U = \hat{U}^{(K)} \dots \hat{U}^{(1)}$. Every operation corresponds to the application of an appropriate Hamiltonian for the specific time τ_k according to $\hat{U}^k = \exp(-i\hat{H}_k\tau_k)$. The Hamiltonians have only two possible forms ($\hbar=1$ throughout)

$$\hat{H}_k^{\text{bs}} = g_k a_{i_k}^\dagger a_{i_k+1} + \text{H. c.}, \quad (3)$$

$$\hat{H}_k^{\text{ps}} = \phi_k a_{j_k}^\dagger a_{j_k}, \quad (4)$$

where indexes $i_k, j_k = 1, \dots, M$ label the resonator modes involved in the k -th operation. Once introduced in the operator \hat{U}^k , the quantities $g_k\tau_k$ and $\phi_k\tau_k$ define, respectively, the beam splitter reflectivity and phase shift associated to the k -th optical element. By applying these building-block operations sequentially, one realizes the complete boson sampling unitary \hat{R}_U . This procedure offers the possibility of implementing boson sampling in any platform capable of generating the above Hamiltonians. In particular, superconducting circuits associate an extraordinary level of control to the required interactions.

In the next section, we show how to implement beam splitting and phase shifting operations in circuit QED systems. We also describe the state preparation and measurement steps that complete the scalable implementation of boson sampling with microwave photons.

BOSON SAMPLING WITH SUPERCONDUCTING CIRCUITS

Boson sampling consists of three fundamental steps: i) initial single-photon state preparation, ii) implementation of the random unitary \hat{R}_U and iii) single-photon detection. Here, we describe the specific circuit design to implement all the necessary operations with microwave photons.

Our proposal consists of a series of high-quality-factor (high-Q) superconducting storage resonators which are coupled to each other by a tunable interaction that can effectively be switched on and off. These resonators are used for storage of the photons that will be processed to carry out the boson sampling algorithm. At the same time, each storage resonator is also coupled to a superconducting qubit to perform the crucial operations required by a boson sampling device (see Figure 1b). While

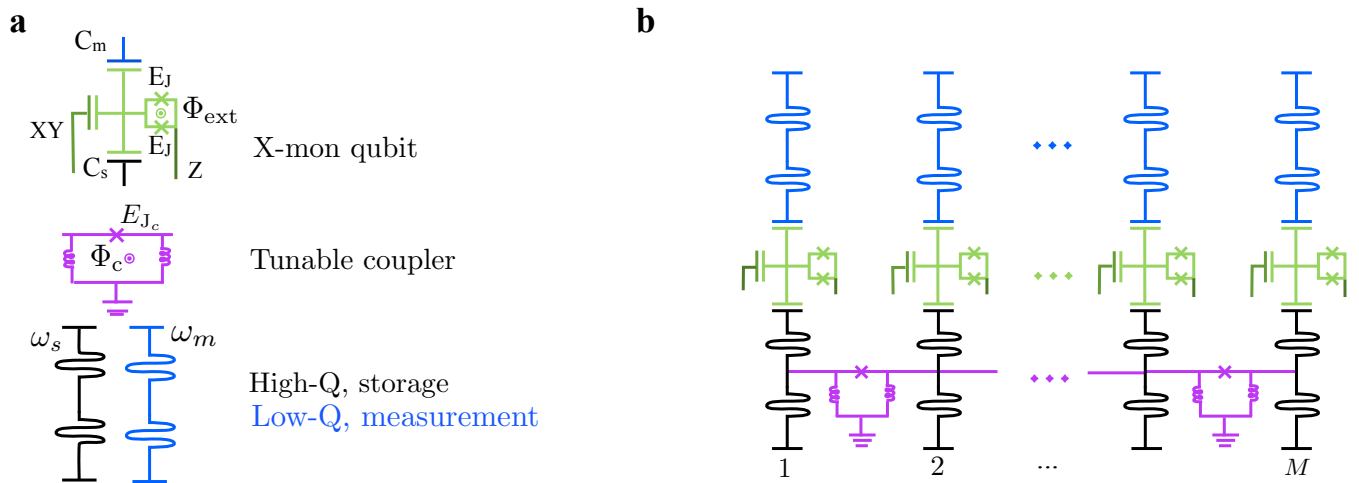


FIG. 1. Boson sampling implementation with microwave photons. a) Quantum electrical circuits required for the physical implementation: (Top) a cross-shaped transmon qubit, also known as Xmon. The qubit is capacitively coupled to the storage (C_s) and measurement resonators (C_m), an XY control line, and inductively coupled to a Z control line that controls the qubit frequency through the external magnetic flux Φ_{ext} . (Middle) A superconducting ring intersected by a Josephson junction, acting as a coupler between neighboring storage resonators. The coupling is fully tunable through the external control flux Φ_c . (Bottom) High-Q storage resonator (black) and low-Q measurement resonator (blue) with characteristic frequencies ω_s and ω_m . b) Experimental proposal for scalable boson sampling using superconducting circuits.

this proposal is largely qubit independent, we have chosen to illustrate it adopting the X-mon qubit [19] since its cross-shaped design allows for both transverse and longitudinal rotations without acting on the resonator themselves. Finally, the design requires the X-mon qubit to be also coupled to a low-Q (measurement) resonator, which will be used to perform quantum non-demolition detection of the photons stored in the storage resonator. We now discuss how to implement all the fundamental operations in superconducting setups:

I) Initial state preparation.- We initialize the qubits, initially far detuned in energy from the storage resonator frequency, in the ground state $|g\rangle$. Then, we coherently drive the first N X-mon qubits through their XY ports to implement a π -pulse that brings the qubits to the excited state $|e\rangle$. This single qubit operation can be done with extremely high fidelity, of around 99.92% as recently reported in a similar system [12, 20]. By tuning the X-mon frequency through the Z qubit control line, we bring the qubits on resonance with the storage resonators for a time t , activating a Jaynes-Cummings interaction of the form

$$H_{\text{JC}} = \omega_s a^\dagger a + \frac{\Omega}{2} \sigma_z + g_s (\sigma^+ a + \sigma^- a^\dagger), \quad (5)$$

where Ω is the qubit frequency, ω_s the storage resonator frequency and g_s is the coupling constant (see Methods). Applying this interaction for a time $t = \pi/g_s$ moves the qubit excitation onto the storage resonator $|e\rangle \otimes |0\rangle \rightarrow |g\rangle \otimes |1\rangle$, creating a single-photon Fock state on the storage resonator. This operation can be performed deterministically and with high efficiency, as shown in

[17]. Interestingly enough, we are not limited to the generation of single-photon states. More complicated states, such as higher-number Fock states [21] and Gaussian states [22], can also be prepared. As we will discuss later on, this would allow the implementation of boson sampling with modified input states in the form required by the quantum simulations of molecular spectroscopy [16].

II) Unitary operation.- In the previous section we showed that any unitary can be written as an appropriate sequence of local Hamiltonians of the form (3) and (4). Beam splitter operations can be simply carried out by bringing two transmission line resonators together. In the confluence of their center conductors, evanescent waves couple the two resonators allowing the photons to tunnel between them. However, their coupling is determined by the fixed geometric arrangement of the resonators, resulting in a static coupling g_{bs} that can not be switched off. In order to make the coupling switchable, different schemes have been proposed theoretically [23, 24] and implemented experimentally [13–15]. All these proposals are based on superconducting rings acting as tunable couplers (cf. Fig. 1a). Switchability relies on a controlled quantum interference between the resonator wavefunctions, that either adds them up or cancels each other out, depending on a control parameter, namely the external magnetic flux Φ_c threading the superconducting ring (see Methods). These tunable interactions have been realized both as qubit-qubit [13] and as resonator-resonator couplers [14, 15], reporting on-off interaction ratios of about 10^4 . Moreover, the switching operation is very fast and takes only a fraction of a nanosecond, that is to say a

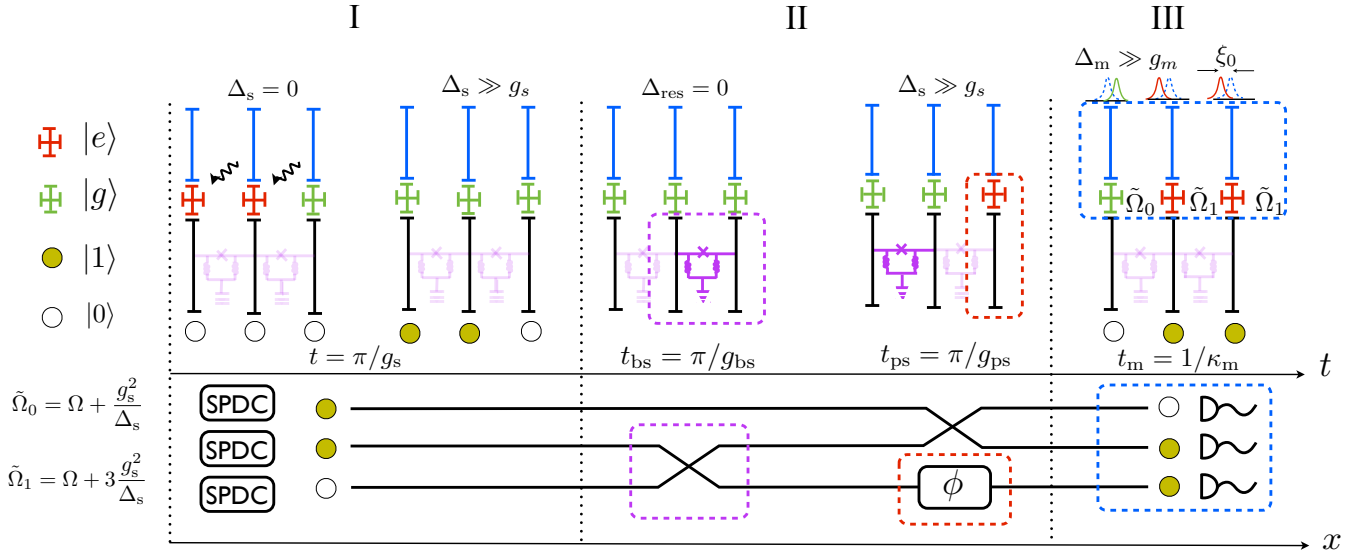


FIG. 2. Pictorial description of microwave boson sampling in a three-mode device, and comparison with its linear optics counterpart. In the optical network photons travel from left to right, passing through the three fundamental steps of boson sampling I) state preparation, II) unitary dynamics and III) detection. The corresponding operations in circuit QED are illustrated in the panel above, where the color code indicates which interaction is currently active. Qubits are depicted in red if in the excited state $|e\rangle$, and green if in the ground state $|g\rangle$. Purple ring coupler are disconnected when faded. The protocol is summarized in Table I.

time scale much faster than the resonator dynamics.

Phase-shifting operations can be implemented by bringing the qubit off-resonance with the storage resonator, in the so-called dispersive regime where $\Delta_s = \Omega - \omega_s \gg g_s$. Under this condition, the qubit induces a state-dependent pull of the resonator frequency of the form

$$H_{\text{dis}} = \left(\omega_s - \frac{g_s^2}{\Delta_s} \sigma_z \right) a^\dagger a + \frac{1}{2} \left(\Omega - \frac{g_s^2}{\Delta_s} \right) \sigma_z, \quad (6)$$

where the effective resonator frequency includes contribution from $\phi = g_s^2/\Delta_s \times \langle \sigma_z \rangle$. As a consequence, the phase accumulated by each photon in the resonator depends on the qubit state, being proportional to $\langle \sigma_z \rangle = \pm 1$ for the excited and ground state, respectively. Assuming that every qubit is in the ground state $|g\rangle$, and equally detuned with respect to its storage resonator, there is no relative frequency shift between resonators. However, relative phases between resonators can be arbitrarily created simply by flipping the corresponding qubit to its excited state $|e\rangle$ and introducing a frequency modification equals to 2ϕ .

Thus, applying a dispersive interaction of the form (6) to a desired qubit-resonator pair for times $t_{\text{ps}} \in [0, \pi/\phi]$ one can introduce arbitrary *relative* phase-shifts between any pair of adjacent storage resonators.

III) Readout.- A very important and delicate step in any superconducting architecture is the measurement protocol. For this reason, we provide two alternative implementations based on distinct physical mechanisms.

The first mechanism consists of mapping the storage resonator state back to the qubits, by inverting the state preparation procedure. This mechanism is supposed to perfectly distinguish between an empty resonator and a resonator occupied by a single microwave photon, as required in the original formulation of boson sampling [2]. Bringing the qubits on resonance with the storage resonators, the interaction in eq. (5) causes Rabi oscillations that swap the boson sampling resonator state $|\psi_{\text{out}}\rangle$ to the qubit [17]. While two or more photons might have bunched together on the same resonator, thus preventing the transfer to the qubit state due to a photon-blockade effect [25], we can postselect this event as we would do in any linear optics implementation. With the aid of a second, low-Q resonator, we perform a quantum non-demolition detection of the qubit state (see Figure 1b). Measuring the transmission of the measurement resonator, we detect with large fidelity whether the qubits are in the ground or excited state, and hence the photon state in the storage resonators [26].

While the measurement described above has similarity with the functioning of a “photodetector” (i.e. discriminate only 0 or 1 microwave photons in the resonator), we devise a second readout mechanism that works as a *high-efficient quantum non-demolition* photon counter. The measurement mechanism is based on qubit-photon logic gates [27]. Within the dispersive regime, where the qubit is detuned by an amount Δ_m , the effective qubit frequency is lifted due to the photons in the storage resonator according to $\tilde{\Omega}_n = \Omega + (2n+1)g_s^2/\Delta$, where n is the

	Step I:	Step II: Unitary operator		Step III:
	Initial state preparation	Beam splitter	Phase sifting	Measurement protocol
Physical system	qubit-storage resonator	resonator-resonator	qubit- storage resonator	qubit- measurement resonator
Hamiltonian	Jaynes-Cummings	beam-splitting	dispersive	dispersive
Relevant parameters	$\Delta_s = 0, t = \pi/g_s$	$\Delta_{\text{res}} = 0, t_{\text{bs}} = \pi/g_{\text{bs}}$	$\phi = g_s^2/\Delta_s, t_{\text{ps}} = \pi/\phi$	$\xi_0 = g_m^2/\Delta_m, t_m = 1/\kappa_m$
Figures of merit	$g_s/2\pi \simeq 150\text{MHz}$	$g_{\text{bs}}/2\pi \simeq 30\text{MHz}$	$\phi \simeq 20\text{MHz}, \kappa_s/2\pi = 1\text{ KHz}$	$\xi_0 = 30\text{MHz}, \kappa_m/2\pi = 20\text{MHz}$

TABLE I. Summary of the microwave boson sampling implementation. For each step of the protocol (columns), we display the key physical systems involved in that step, the Hamiltonian ruling the system dynamics, as well as the relevant parameters and their figures of merit. In step I, qubit and storage resonator interact of resonance via Jaynes-Cummings Hamiltonian. In step II, storage resonators are coupled on resonance via beam-splitter interaction for the purposes of beam splitting operations, while an off resonance, dispersive interaction with the qubit implements relative phase shifts. In step III, off resonance dispersive interaction, this time with the measurement resonator, is used for quantum non-demolition detection.

number of photons on the storage the resonator. Then, by sending coherent microwave signals at the different frequencies of the qubit $\tilde{\Omega}_n$, we perform a π -rotation on the qubit, contingent on the storage resonator state $|n\rangle$: when the driving microwave hits the qubit at its resonant frequency, we flip the qubit state $|g\rangle \rightarrow |e\rangle$, which will, in turn, create a displacement of the measurement resonator frequency [26]. By tracking the transmission on the measurement resonator, we can determine the number of photons n in the storage resonator in at most n trials. As far as boson sampling is concerned this would normally correspond to one or two attempts to measure the resonator. Each readout can be performed with efficiency of about 90% [27] and, since the measurement is non-demolition, one can repeat the measurement many times to exponentially reduce the probability of failure. The latter readout scheme represents a remarkable feature of our microwave setup that is absent, in its deterministic form, in linear optical setups. This results is very relevant for the realization of boson sampling experiments that require counting more than one photon per mode, as is the case for modified boson sampling protocols with initial Gaussian states [4, 16].

To illustrate our proposal, we present in Figure 2 a pictorial comparison of a three-mode boson sampling implementation with superconducting circuits and the original linear optical network. A summary of the whole microwave boson sampling protocol can be found in Table I, where we present the most relevant parameters together with their experimental benchmarks.

GENERALIZED BOSON SAMPLING WITH SUPERCONDUCTING CIRCUITS

At various points during the description of the proposed microwave setup, we observed that the superconducting design allows not only the implementation of all passive linear elements, but also several additional operations. This flexibility represents a necessary condition to realize many generalized versions of the boson sampling problem [4, 6]. Implementations with Gaus-

sian states require, for example, the ability of preparing two-mode squeezed states and to perform parity measurements. A particular role is played by the proposal in Ref. [16], which constitutes the first practical application of boson sampling and connects it to molecular spectroscopy. Here, we describe how the required operations of displacement, squeezing and photon-number discrimination are achievable with microwave photons.

Consider the very same device presented to tune the resonator-resonator couplings. The specific form of the interaction in eq. (3) is obtained in the rotating wave approximation starting from the more accurate form $H_{\text{int}} = g_k(\Phi_c)(a_k^\dagger + a_k)(a_{k+1}^\dagger + a_{k+1})$. As detailed in the methods section, when the external magnetic flux through the coupler Φ_c oscillates at the appropriate frequency $\omega_c = \omega_k + \omega_{k+1}$, the interaction effectively produces two-mode squeezing in the frame rotating at the coupler frequency. Simultaneously, a displacement operation can be straightforwardly introduced by simply driving the storage resonator itself. The combined action of displacement and squeezing is interpreted as the required state preparation step of modified boson sampling setups [4, 16]. The other essential requirement is the ability of determining the parity or counting the number of photons in a resonator. This operation has already been described in the previous section: in essence, we exploit the nearby qubit to check a single occupation number of the storage resonator at a time, effectively implementing a quantum non-demolition photon counter. By virtue of the suggested protocol, our proposal constitutes, to the best of our knowledge, the first scalable implementation of any practical application of boson sampling.

DISCUSSION

To address the feasibility of our proposal, we have to understand how the requirements on the single operation affect the overall scalability. First of all, the number of consecutive beam splitter or phase shifting operations to be performed increases with the number of modes M . In

general, one needs $\mathcal{O}(M^2)$ operations [18], but we observe that in our setup, like in the optical counterpart, $\mathcal{O}(M)$ operations can be implemented simultaneously. This relaxes the requirement on the quality factor of the storage resonators, since a resonator lifetime proportional to M is sufficient. How many operations can we perform, and therefore how many modes can we consider, before the microwave photons are lost? Loading and measuring the resonator is performed only once per run, while a typical operation consisting of a beam splitter followed by a phase shifter requires a time $(t_{\text{bs}} + t_{\text{ps}}) \simeq 0.3\mu\text{s}$ (see values reported in Table 1). This time has to be compared with the storage resonator lifetime, which would probably be the limiting factor to run a successful experiment. High finesse coplanar waveguides resonators with quality factors above one million have been reported [28, 29], yielding cavity decay rates $\kappa \simeq 2\pi \times 1$ KHz corresponding to a cavity lifetime $t_\kappa = 150\mu\text{s}$. Thus, one could implement a total number of operations $t_\kappa/(t_{\text{bs}} + t_{\text{ps}}) \simeq 500$ before the photons are lost.

Since boson sampling is believed to be hard for $N \sim \sqrt{M}$, we can successfully manipulate ~ 20 photons. At the same time, the probability of correctly preparing and detecting all the N single photons diminishes exponentially in any non error-corrected architecture, and superconducting circuits are not an exception. However, the remarkable fidelities $\mathcal{F} \simeq 99.9\%$ achieved in generating and measuring single photon Fock states [22] demonstrate that the superconducting technology is already mature to successfully implement boson sampling with $N \sim 20$ photons. This size is at the edge of what is tractable on a classical supercomputer [2, 30] and, therefore, we are confident that the first post-classical computation is within experimental reach with today's technology.

In conclusion, we propose a novel architecture to overcome the limitations exhibited by the linear optical and ion trap implementations of boson sampling. We start from the observation that any photonic network can be decomposed in a sequence of elementary operations generated by two kind of Hamiltonians alone. Then, we suggest to realize the bosonic modes by identical microwave resonators that are coupled to each other with tunable strength. For each resonator, a superconducting X-mon qubit provides the access needed to perform state preparation and measurement. A consequence of the proposed design is that other non-linear operations, like those introduced in recent works on generalized versions of boson sampling, are readily implementable. In particular, squeezing operations and photon counter measurements are now available to realize the first practical application of boson sampling in the context of molecular spectroscopy.

METHODS

On- and off-resonant regimes

The X-mon qubit used in this proposal works as a split-transmon [31], where a dc-SQUID acting as a tunable Josephson junction induces fast changes in the qubit frequency (see Fig. 1a, top for the circuit design, and the definition of the relevant quantities considered below). More specifically, the qubit frequency is given by $\Omega = \sqrt{8E_J(\Phi_{\text{ext}})E_C}$, where $E_C = e^2/2C_J$ is the capacitive energy of the qubit, and $E_J(\Phi_{\text{ext}}) = 2E_J \cos(2\pi\Phi_{\text{ext}}/\Phi_0)$ is the effective Josephson energy. Through variations of the external magnetic flux Φ_{ext} , one can change Ω to the desired frequency range. More precisely, we are interested in bringing the qubit on and off resonance with the storage resonators which accounts for frequency change of a few GHz. Such a frequency change can be done in a few nanoseconds, without altering the qubit lifetime, of the order of tens of microseconds.

Finally, the coupling constant that characterizes the interaction between qubits and resonators, valid for both storage and measurement resonator, is given by [32]

$$g_{s,m} = \frac{C_{s,m}}{C_{s,m} + C_J} \sqrt{\frac{\omega_{s,m}}{cL}}, \quad (7)$$

where $C_{s,m}$ is the capacitive coupling of the qubit to the storage (measurement) resonator, L is the resonator length, and c the capacitance per unit length.

Beam splitting and two-mode squeezing operations

The more generic resonator-resonator interaction can be written in the interaction picture as [23]

$$H_{\text{int}} = g(\Phi_c)(a_k^\dagger + a_k)(a_{k+1}^\dagger + a_{k+1}), \quad (8)$$

where a_k (a_k^\dagger) is the annihilation (creation) operator of the k -th resonator, satisfying canonical commutation rules $[a_k, a_{k'}^\dagger] = \delta_{kk'}$ and $g(\Phi_c)$ is a flux-dependent coupling constant of the form

$$g(\Phi_c) = g_{\text{bs}} \cos(2\pi\Phi_c/\Phi_0). \quad (9)$$

For the purposes of implementing beam splitting operations over identical resonators ($\omega_k = \omega_{k+1} \forall k$), one just needs a step-function dependence of static external fluxes, since Hamiltonian (8) follows immediately after the rotating wave approximation [23]. More precisely, for switching on a 50/50% beam splitter interaction one applies an external flux $\Phi_c = n\Phi_0$ for a time $t = \pi/g_{\text{bs}}$, while an externally applied flux at $\Phi_c = (n + 1/2)\Phi_0$ will switch it off. On the other hand, the same interaction (8) can effectively generate squeezing operations when the applied flux oscillates at the appropriate frequency. In particular, for an external flux $\Phi_c(t) =$

$\Phi_c \cos((\omega_k + \omega_{k+1})t)$, and invoking the Anger-Jacobi expansion, one can go to a new rotating frame that yields the effective Hamiltonian $H_{\text{int}} = g_{\text{bs}}(a_k^\dagger a_{k+1}^\dagger + \text{H. c.})$.

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AUTHOR CONTRIBUTIONS

B.P. and A.A.-G envision the project. B.P. and G.G.G. developed the protocol. B.P., G.G.G, J.H. and A.A.-G. were involved in the research process, in discussions and in writing the manuscript.

COMPETING FINANCIAL INTERESTS

The authors declare no competing financial interests.

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