



# Phase-induced topological superconductivity in a planar heterostructure

Omri Lesser<sup>a</sup>, Andrew Saydjari<sup>b</sup>, Marie Wesson<sup>c</sup>, Amir Yacoby<sup>b,c,1</sup>, and Yuval Oreg<sup>a,1</sup>

<sup>a</sup>Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 7610001, Israel; <sup>b</sup>Department of Physics, Harvard University, Cambridge, MA 02138; and <sup>c</sup>John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138

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**Topological superconductivity in quasi-one-dimensional systems is a novel phase of matter with possible implications for quantum computation. Despite years of effort, a definitive signature of this phase in experiments is still debated. A major cause of this ambiguity is the side effects of applying a magnetic field: induced in-gap states, vortices, and alignment issues. Here we propose a planar semiconductor–superconductor heterostructure as a platform for realizing topological superconductivity without applying a magnetic field to the two-dimensional electron gas hosting the topological state. Time-reversal symmetry is broken only by phase biasing the proximitizing superconductors, which can be achieved using extremely small fluxes or bias currents far from the quasi-one-dimensional channel. Our platform is based on interference between this phase biasing and the phase arising from strong spin–orbit coupling in closed electron trajectories. The principle is demonstrated analytically using a simple model, and then shown numerically for realistic devices. We show a robust topological phase diagram, as well as explicit wavefunctions of Majorana zero modes. We discuss experimental issues regarding the practical implementation of our proposal, establishing it as an accessible scheme with contemporary experimental techniques.**

topological phases of matter | Majorana zero modes | topological superconductivity

The quest for discovering novel phases of matter has seen rapid development in the past few decades, with the advent of topological quantum matter (1–5). Phases that would be equivalent under Landau’s order parameter paradigm were found to be distinguished by topological properties. The earliest and perhaps most salient phase characterized by its topology is the quantum Hall effect (6), where the chiral edge modes signal the nontrivial bulk topology (7, 8). It was later realized that a similar phenomenon can take place with a zero net magnetic flux (9), which opened the door to the field of Chern insulators.

Shortly after these important discoveries, the connection to superconductivity (SC) was made, by the concept of topological SC. This novel phase of matter supports Majorana zero modes (MZMs), which are predicted to have non-Abelian exchange statistics (10, 11). It has been vigorously studied in recent years, following early ideas in one (12) and two (13) spatial dimensions. Further theoretical studies advanced the field from toy models to practical implementations (14), fueled by the potential application of the exotic MZMs in quantum computation (11, 15, 16).

A prominent Majorana platform is hybrid superconductor–semiconductor nanowires (17, 18), where a Zeeman field combined with strong spin–orbit coupling (SOC) drives a proximitized nanowire into a topologically nontrivial phase. Following the theoretical proposals, several tunneling experiments (19, 20) observed zero-bias conductance peaks as possible signatures of MZMs. The early results triggered many subsequent studies, including on current-biased (21) and disordered (22, 23) nanowires and carbon nanotubes (24–26) as the one-dimensional (1D) platform. Other important Majorana realiza-

tion platforms include quantum wells subjected to an in-plane magnetic field (27), topological insulator–superconductor heterostructures (13), semiconductor–ferromagnet heterostructures (28, 29), chains of magnetic adatoms on spin–orbit-coupled superconductors (30, 31), iron-based superconductors (32, 33), and, most recently, full-shell proximitized nanowires (34, 35).

A different route, first taken in two important theoretical works (36, 37), is based on planar semiconductor–superconductor heterostructures. These studies proposed using a phase-biased Josephson junction, forming a quasi-1D geometry, with an applied in-plane magnetic field. Such a planar device is a highly attractive platform for the experimental realization of topological SC, as it is less delicate than nanowires, does not require strict magnetic field alignment, and is easier to fabricate. The theoretical proposals triggered two recent experiments that indeed reported zero-bias conductance peaks which may indicate the existence of MZMs (38, 39). However, a drawback of this platform is the need to apply an appreciable magnetic field to the device. The applied magnetic field diminishes SC and could give rise to magnetic impurity states, hindering the detection of MZMs.

In this manuscript, we propose a route to realizing topological SC in a planar geometry without applying a magnetic field in the Josephson junction. Previous proposals achieved this by using supercurrents to break time-reversal symmetry; however, the large currents required and fringing fields in the junction complicate implementation (40). Our proposal relies solely on phase biasing the proximitizing superconductor, a possibility that was recently pointed out (41). The approach is based on engineering the geometry such that the topological Aharonov–Casher phase

## Significance

**The practical realization of Majorana zero modes in quasi-one-dimensional topological superconductors is greatly hindered by the need to apply strong magnetic fields. This study proposes a way to engineer these exotic states using only superconducting phase bias, which requires negligible magnetic fields or currents. The proposed device is experimentally accessible and robust, as we show by comprehensive theoretical modeling. Furthermore, it has the potential of providing substantially cleaner experimental signatures of Majorana zero modes than the currently available platforms, paving the way to building a topological qubit.**

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<sup>1</sup>To whom correspondence may be addressed. Email: yacoby@physics.harvard.edu or yuval.oreg@weizmann.ac.il.

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(42), induced by the SOC, constructively interferes with the SC phase winding (13, 41, 43), giving rise to a zero-energy state. We would like to emphasize that the role of the phase bias in our proposal and in Fu and Kane's (13) proposal is different. In Fu and Kane's proposal, the topological phase results from the interplay between the surface of a 3D topological insulator and a superconductor, even in the absence of phase bias. The role of the phase bias is to form a discrete vortex supporting an MZM. Here the phases themselves drive the system into a topological phase in a conventional material platform with strong SOC. Our geometry also benefits from eliminating unbound trajectories as in the case of zigzag junctions (44). Not only does this proposal eliminate the complications of magnetic fields in the junction coexisting with the MZMs, but it is also accessible with current materials and fabrication techniques.

### Minimal Model and "Sweet Spot"

We shall now demonstrate the possibility of realizing MZMs in a magnetic field-free planar geometry. To this end, we develop a toy model that supports a "sweet spot" with perfectly localized MZMs (45). This is analogous to tuning the Kitaev chain to zero chemical potential and equal hopping and pairing amplitudes, which makes the edge modes perfectly localized in a single site (12).

The basic building block of our model is a ring composed of  $N \geq 3$  sites; see Fig. 1A. Electrons can hop from each site to its nearest neighbors through a spin-orbit-coupled medium, and each site  $n$  is proximitized by an SC with a different phase  $\phi_n$ . The ring is described by the tight-binding Bogoliubov-de Gennes (BdG) Hamiltonian

$$H = \sum_{n=1}^N \left( t e^{i\lambda\sigma_z} c_n^\dagger c_{n+1} - \frac{\mu}{2} c_n^\dagger c_n + \text{H.c.} \right) + \sum_{n=1}^N \left( \Delta e^{i\phi_n} c_{n,\uparrow}^\dagger c_{n,\downarrow}^\dagger + \text{H.c.} \right), \quad [1]$$

where  $\sigma$  labels the spin degree of freedom,  $t$  is the hopping amplitude,  $\lambda$  is the SOC angle,  $\mu$  is the chemical potential, H.c.

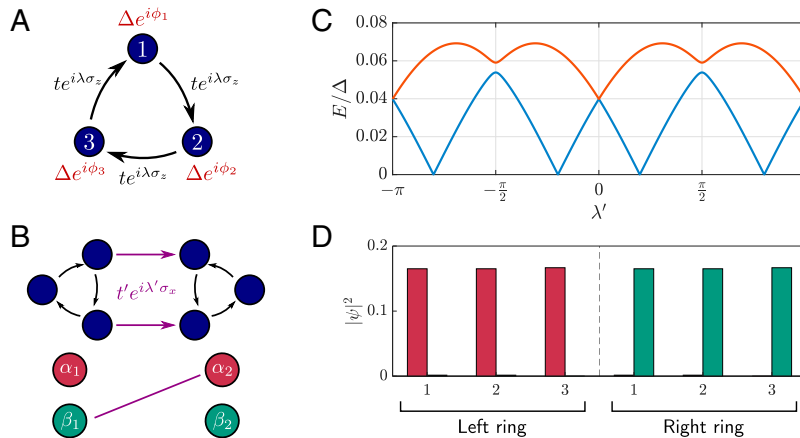
is the Hermitian conjugate, and  $\Delta$  is the SC pairing potential. The gauge transformation  $c_n \rightarrow c_n e^{i\phi_n/2}$  eliminates the phases from the SC terms, in exchange for adding the complex amplitude  $e^{i(\phi_{n+1}-\phi_n)/2}$  to the hopping term from site  $n+1$  to site  $n$ .

It is known that SC phase winding is necessary in order to obtain a zero-energy state in such a model (43). For simplicity, we assume that  $\phi_n = n\varphi$ , where  $\varphi = 2\pi m/N$  with  $m \in \mathbb{Z}$ , that is, an integer number of vortices. With this choice,  $\phi_{n+1} - \phi_n = \varphi$  is constant, so the model becomes translationally invariant and can therefore be analyzed in momentum space. The Bloch Hamiltonian reads

$$H = \sum_{k,\sigma} \left[ 2t \cos \left( k + \frac{\varphi}{2} + \lambda\sigma \right) - \mu \right] c_{k\sigma}^\dagger c_{k\sigma} + \sum_k \left( \Delta c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \text{H.c.} \right), \quad [2]$$

where  $k = 2\pi/Nq$ ,  $q = 0, 1, \dots, N-1$ . As we show in *SI Appendix, section I*, it is possible to tune the model's parameters such that two zero-energy states appear: a pair of MZMs, delocalized around the ring. The key ingredient of the model is the interplay between the gauge-invariant topological Aharonov-Casher phase,  $3\lambda$  (arising due to electron and hole trajectories circulating the ring), and the SC phase winding.

Having established the possibility of realizing MZMs in a ring, the next step is coupling two such rings, as shown in Fig. 1B. Each ring hosts two MZMs, and our goal is to couple the rings such that one MZM in each ring remains uncoupled, and the other two MZMs are gapped out, as illustrated at the bottom of Fig. 1B. Since the bare MZMs are delocalized throughout their rings, the only way to achieve this goal is by interference of trajectories (46), and therefore the two rings must be coupled via more than one link. In the coupling form suggested in Fig. 1B, the rings are



**Fig. 1.** Toy model for realizing a "sweet spot" relying on SC phase bias only. (A) Illustration of the ring described by Eq. 1 for  $N = 3$  sites, each connected to its nearest neighbors by a hopping amplitude  $t$  through a spin-orbit-coupled medium characterized by the angle  $\lambda$ . Each site is proximitized by a superconductor with pair potential  $\Delta$  and a different phase  $\phi_n$ . The parameters controlling the ring's behavior are tuned such that each ring contains two overlapping MZMs,  $\alpha$  and  $\beta$ . (B) Two rings are coupled at two points (top). If only  $\beta_1$  and  $\alpha_2$  are coupled, those states are gapped out, leaving the zero-energy states  $\alpha_1, \beta_2$  intact and localized at the left and right rings, respectively (bottom). (C) Energies of the lowest-lying (blue) and second-lowest-lying (orange) states in the system composed of two coupled rings. For certain values of  $\lambda'$ , a zero-energy state appears, with a finite gap to the second-lowest state. At  $\lambda' = 0, \pm\pi$ , we observe spin degeneracy. (D) Zero-energy Majorana wavefunctions at one of the special values of  $\lambda'$ , each state localized in a different ring.

connected at two points via spin-orbit-coupled links of amplitude  $t' e^{i\lambda' \sigma_x}$ .

By choosing the SC phases at the two rings and controlling the inter-ring SOC angle  $\lambda'$ , it is possible to tune into a “sweet spot” where one MZM in each ring remains intact. We show this explicitly in *SI Appendix, section I* by projecting the coupling Hamiltonian to the low-energy subspace and making sure that one MZM in each ring is left uncoupled. This situation is demonstrated in Fig. 1C, where we plot the energies of the lowest and second-lowest states as a function of  $\lambda'$ . At special values of  $\lambda'$ , the lowest energy is zero, and the second-lowest energy is finite, implying the existence of two MZMs, each localized in a different ring, with a gap to excitations. If we now concatenate these double-ring building blocks on the plane, the localization length of the Majorana edge modes will be one ring independent of the overall length of the chain. We have therefore achieved a topological superconducting phase by tuning the three superconducting phases, without the Zeeman effect. The sweet spot is obtained by setting  $\mu$ ,  $\Delta$ ,  $\lambda$ , and  $\lambda'$  to proper values.

### Realistic Model

How can one realize a phase similar to the “sweet spot” in a realistic experiment? The key point is using a geometry that supports closed orbits with a nonzero Aharonov-Casher phase (42) and SC phase winding (41). Here we demonstrate this idea using a planar semiconductor-superconductor heterostructure and analyze the topological phase diagram as a function of easily controlled parameters—the SC phases and the chemical potential.

We consider the unit cell geometry depicted in Fig. 2. The system is made of a 2D electron gas (2DEG) with Rashba SOC, partially covered by SCs with fixed phases. The main difference between this configuration and the original proposals that rely on a Zeeman field (36, 37) is the addition of a third SC with phase control. The nonstraight features, characterized by the lengths ( $W_B, L_B$ ), will help stabilize the topological phase by increasing the energy gap (along the lines of ref. 44). In what follows, we will show that, when the SC phases wind, the system can be driven into a topological SC phase akin to the one realized in the toy model, even at zero applied Zeeman field.

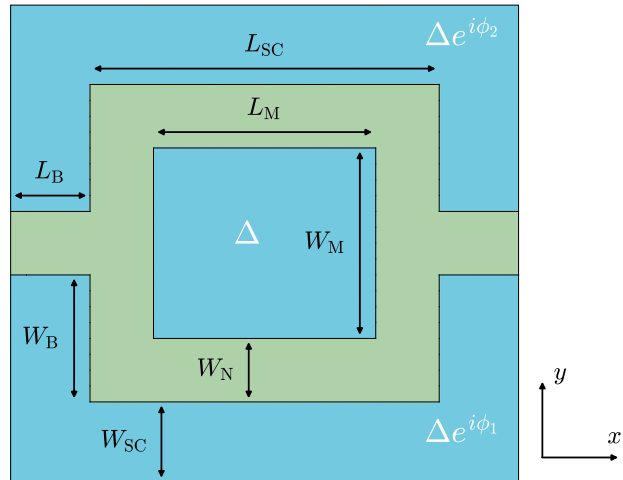
The heterostructure is described by the Hamiltonian

$$H = \left[ \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2) + \hbar\alpha (\sigma_y k_x - \sigma_x k_y) - \mu \right] \tau_z + \text{Re}\Delta(x, y)\tau_x + \text{Im}\Delta(x, y)\tau_y, \quad [3]$$

where  $k_x, k_y$  are the momenta along the  $x, y$  directions,  $m^*$  is the effective electron mass,  $\alpha$  is the Rashba SOC parameter,  $\mu$  is the chemical potential,  $\Delta$  is the local superconducting pairing potential, and  $\tau$  and  $\sigma$  Pauli matrices act in particle-hole and spin spaces, respectively. The magnitude of  $\Delta(x, y)$  is taken to be a constant  $\Delta$  in the proximitized regions, and zero in the nonproximitized regions. The SC phases are zero at the middle SC,  $\phi_1$  at the bottom SC, and  $\phi_2$  at the top SC, as indicated in Fig. 2. The corresponding Nambu spinor is  $\vec{\Psi} = (\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger)^T$ , where  $\psi_s$  annihilates an electron of spin  $s$  along the  $z$  axis.

The system is assumed to be finite along the  $y$  direction, and the unit cell is repeated along the  $x$  direction. For the numerical simulations, the Hamiltonian Eq. 3 is discretized on a square lattice (unless specified otherwise, the lattice spacing is  $a = 10$  nm). Further details on the tight-binding model are given in *SI Appendix, section II*. We note that, while the geometry pro-

\*The  $\sigma_x$  Pauli matrix appears in the inter-ring SOC because of the choice made in the original Hamiltonian Eq. 1 of taking all intra-ring SOC terms proportional to  $\sigma_z$ , which is a local rotation of the spin.



**Fig. 2.** Unit cell of the proposed geometry for realizing topological SC in a planar heterostructure using phase bias only. Blue regions are covered by SCs, whereas the green region is an uncovered 2DEG. The SC phases are zero at the middle SC,  $\phi_1$  at the bottom SC, and  $\phi_2$  at the top SC. The geometric parameters are indicated on the schematic. When  $W_B > W_N$ , straight trajectories that do not encounter SCs at all are eliminated, thus facilitating a large topological gap (44). The unit cell is repeated along the  $x$  direction.

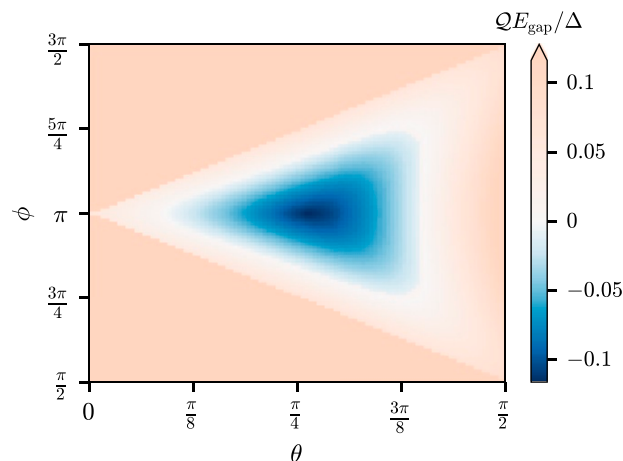
posed in Fig. 2 is the focus of this study, it is merely an example of the general concept. As long as the geometry supports trajectories with a discrete vortex and a spin-orbit phase, it is expected to work as well. We have numerically demonstrated a topological phase transition in several similar geometries.

### Topological Phase Diagram

In this section, we present the full topological phase diagram of the proposed system. The Hamiltonian Eq. 3 belongs to symmetry class D (47–49), with the particle-hole operator being  $\mathcal{P} = \tau_y \sigma_y \mathcal{K}$  ( $\mathcal{K}$  is complex conjugation). Enhancement to the BDI class, as in the case of a single planar Josephson junction (50), is generally prevented by the existence of two distinct phase differences, and only occurs at fine-tuned conditions. In (quasi) one dimension, systems in symmetry class D possess a  $\mathbb{Z}_2$  topological invariant that we label  $\mathcal{Q}$ , which takes the values  $+1, -1$  for the trivial and topological phases, respectively. We calculate the invariant by attaching two leads to a finite-size system and invoking the scattering formula  $\mathcal{Q} = \det r$ , where  $r$  is the reflection matrix from one of the leads to itself (51). The scattering calculations are performed using the Kwant software package (52).

Apart from the topological invariant, the key property of the system is the energy gap. To find the gap, we impose periodic boundary conditions along  $x$ , such that the momentum  $k_x$  is a good quantum number. We then densely scan the Brillouin zone and calculate the lowest energy at each value of  $k_x$ .

Fig. 3 shows the resulting phase diagram as a function of the phase differences  $\theta = (\phi_1 - \phi_2)/2$  and  $\phi = (\phi_1 + \phi_2)/2$ . The system only supports a topological phase when the SC phases wind, which is evident from the phase diagram:  $\mathcal{Q} = -1$  only in certain regions inside the triangle  $\pi - \theta < \phi < \pi + \theta$  (for  $0 \leq \theta \leq \pi/2$ ). This triangle sets the optimal phase boundaries we can expect in such a system (41, 43), and the topological region we obtain here is not far from that. Furthermore, we get a substantial topological gap of over 10% of the induced SC gap  $\Delta$ . Such a large energy gap has important implications for making the MZM localization length short, and for enabling reliable experimental detection. The maximal value of the gap is obtained near the center of the topological region.



**Fig. 3.** Topological phase diagram of the model Eq. 3, showing the topological invariant  $\mathcal{Q}$  which is  $-1$  ( $+1$ ) in the topological (trivial) phase, multiplied by the energy gap. The phase diagram is shown as a function of  $\theta = (\phi_1 - \phi_2)/2$  and  $\phi = (\phi_1 + \phi_2)/2$ . The physical parameters are  $\mu = 0.26$  meV,  $\alpha = 23$  meV  $\cdot$  nm, and the geometric parameters are  $L_{SC} = 320$  nm,  $W_{SC} = 50$  nm,  $L_B = 50$  nm,  $W_B = 80$  nm,  $L_M = 240$  nm,  $W_M = 120$  nm,  $W_N = 40$  nm. See ref. 53 for an interactive version of this figure.

In this structure, closed trajectories in which the electron scatters off all three SCs seem vital for the existence of the topological phase (41). They are present also when the discontinuous rectangles of the middle SC are connected by a thin SC, thus becoming continuous. Then, normal tunneling or crossed Andreev tunneling through the thin segment will enable such trajectories. We have verified numerically that this is indeed possible, but, in our checks, this came at the price of lowering the topological gap. Notice, however, that, in a completely straight geometry, that is,  $W_B = 0$ , it is impossible to obtain a topological phase—the required closed orbits do not exist.

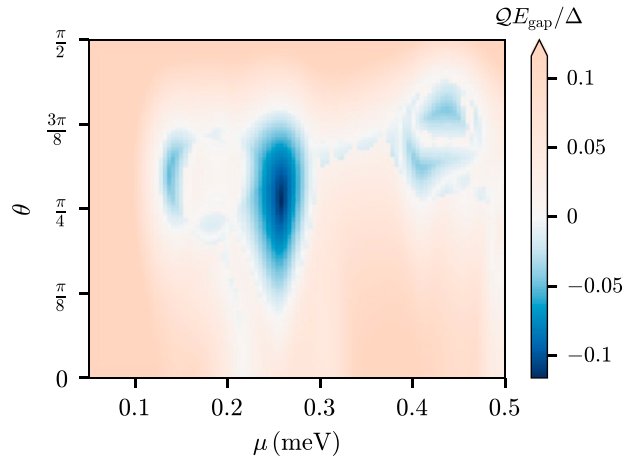
To demonstrate the robustness of the topological phase, we examine its stability to variations in the chemical potential and phase bias. In Fig. 4, we show the topological phase diagram as a function of  $\mu$  and  $\theta$ , fixing  $\phi = \pi$ . We find that the topological phase persists for an appreciable range of parameters, indicating that no delicate fine-tuning is necessary for the system to support MZMs. Stability to such variations is of paramount practical importance, as elaborated in *Experimental Considerations*.

The geometric parameters are chosen here according to the rule of thumb  $L\Delta \approx \alpha$  (41), where  $L$  is a typical length in the uncovered 2DEG region. Given different 2DEG and SC material, one should tune the lengths to approximately match this condition [see *SI Appendix, section III* and the interactive figure (53)].

### Real-Space Analysis and Majorana Wavefunctions

In the previous section, we established the existence of a topological phase by calculating the  $\mathbb{Z}_2$  invariant. We now turn our focus to the real-space hallmark of the topological phase, which is the existence of localized zero-energy Majorana end states. These states are the truly tangible manifestation of the topological nature of the phase, and, experimentally, they reveal themselves as zero-bias conductance peaks in tunneling measurements (19, 20, 27, 38, 39).

A typical MZM wavefunction is shown in Fig. 5, both as a 2D heat map and as a 1D curve (integrated along the transverse direction  $y$ ). The parameters are chosen well within the topo-



**Fig. 4.** Topological phase diagram of the model Eq. 3, as a function of the chemical potential  $\mu$  and the phase difference  $\theta$ , for  $\phi = \pi$ . The parameters are the same as in Fig. 3.

logical phase, as inferred from Fig. 3. As we expect, two states of near-zero energy appear in this regime, localized at opposite edges of the device. The MZM wavefunctions are concentrated a bit more on the normal 2DEG regions than on the SC regions, although their noticeable leakage into the SC region indicates a strong proximity effect.

Remarkably, the Majorana states are localized over one unit cell, which is about  $0.5 \mu\text{m}$  long. A smaller unit cell for the same spin-orbit energy would deviate from the optimal geometry, leading to a smaller topological gap. This localization length also agrees with the continuum approximation  $\xi \approx \hbar v_F / E_{\text{gap}}$ , where  $v_F$  is the Fermi velocity. The relation between the localization length and the energy gap is further demonstrated in Fig. 6A, by calculating both of them as a function of  $\theta$  inside the topological phase.

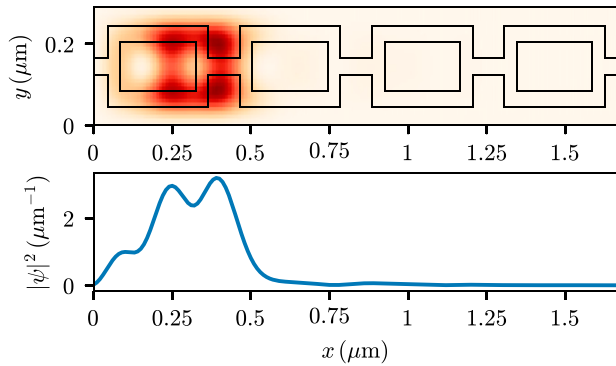
We close this section with another explicit demonstration of the topological phase transition. Fig. 6B shows the evolution of the lowest-lying energies as a function of  $\theta$  in a finite system of six unit cells.<sup>†</sup> The energy gap closes and reopens, leaving a pair of MZMs bound to zero energy. This signals the transition from the trivial to the topological phase. The topological gap is maximized near  $\theta = \pi/4$ , and then closes and reopens again near  $\theta = 3\pi/8$ , this time leaving no in-gap states, indicating that the system returns to the topologically trivial phase. Notice the agreement between this real-space calculation and the  $\phi = \pi$  cut of the phase diagram shown in Fig. 3.

### Experimental Considerations

This proposal for phase-only topological SC can apply to a general class of material platforms: large spin-orbit semiconducting quantum wells such as HgTe, InAs, and InSb which have well-developed fabrication procedures (54–56). While induced SC has been demonstrated in all three platforms, we focus on HgTe below due to its high mobility ( $\mu = 100 \times 10^3 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$  to  $800 \times 10^3 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$ ) and lack of surface states.

The minimum feature sizes of the proposed geometry (Fig. 2) can be achieved by standard electron beam lithography ( $\geq 30$  nm). See the legend of Fig. 7 for a brief outline of the proposed fabrication procedure. Practical geometry constraints are also placed by the material. The width  $W$  between superconducting contacts must be smaller than the coherence length

<sup>†</sup> Notice that  $\theta$  is chosen for convenience, and a similar plot as a function of  $\mu$  or  $\phi$  may also be obtained.



**Fig. 5.** Real-space simulation of the Hamiltonian Eq. 3 with the geometry proposed in Fig. 2, in the topological phase. (Top) Wavefunction of the zero-energy Majorana state localized at the left edge. The wavefunction is almost completely localized at the first unit cell. The black borders indicate the boundary between the SC and the 2DEG. Notice that the MZM wavefunction has significant support beneath the SCs; we find that such behavior occurs for a wide range of parameters, but not always. (Bottom) The above wavefunction integrated over  $y$  shows exponential decay along  $x$ . An exponential fit yields a localization length of  $\xi_M \approx 0.5 \mu\text{m}$ . The simulation included 10 unit cells, and, for clarity, only the 4 left-most ones are shown. The other MZM is localized at the right edge of the device. The parameters used are the same as in Fig. 3, with  $\phi = \pi$ ,  $\theta = \pi/4$ .

of the induced SC in the Josephson junction which is limited to  $\zeta_T \approx \hbar v_F / 2\pi k_B T$ . While the induced coherence length provides a loose bound on the junction width to  $\leq 50 \mu\text{m}$  at 20 mK, this bound quickly becomes strict at higher temperatures. Further, in order to work in the quasi-ballistic regime, the mean free path  $l_e$  must be at least on the order of  $W$ . For HgTe, we have  $l_e \approx 1.6 \mu\text{m}$  for  $\mu \approx 100 \times 10^3 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$  and  $n \approx 1 \times 10^{12} \text{ cm}^{-2}$ . Since there is no competing drive for larger junction widths, as in the case of the in-plane field proposal (36, 37), this upper bound need not be saturated, thus suggesting our proposal will be more robust to lower-quality semiconducting materials.

With these upper and lower bounds from the material in mind, we still have to determine the optimal operational dimensions of the device. Dimensional analysis suggests that a typical coherence length set by  $\alpha/\Delta$  should be of the order of the typical lengths of the device  $L$ . Ref. 41 analyzed a simplified tight-binding model of a related (nonplanar) system, and showed that this rule of thumb indeed leads to an optimal situation, with a maximal topological region in the  $\theta$ - $\phi$  plane of Fig. 3. For fixed  $\Delta$ , this rule of thumb,  $L\Delta \approx \alpha$ , implies that, for larger SOC, larger unit cells are required to achieve the largest topological gap. In contrast to fully 1D proposals, here the confinement of the MZM is set by the size of the unit cell, rather than just the topological gap as is expected in a continuum setting. For moderate  $\alpha = 23 \text{ meV}\cdot\text{nm}$  with the superconducting gap of aluminum, this condition fixes  $L \approx 1 \mu\text{m}$ . For fixed  $\alpha$ , increasing the superconducting gap shortens this length scale. Advantageously, HgTe growth can be tuned from negligible toward the very large spin-orbit splitting up to 30 meV (57, 58), with  $\alpha$  as large as 120 meV·nm, which can be matched to the superconductor used to balance fabrication limitations on dimension and optimize confinement. Notice that, even for the case of Al, we saturate lithographic precision at  $\alpha \approx 10 \text{ meV}\cdot\text{nm}$ , where  $E_{SO} \approx 1.5 \text{ meV} \geq \Delta$ , suggesting extraordinary confinement is possible.

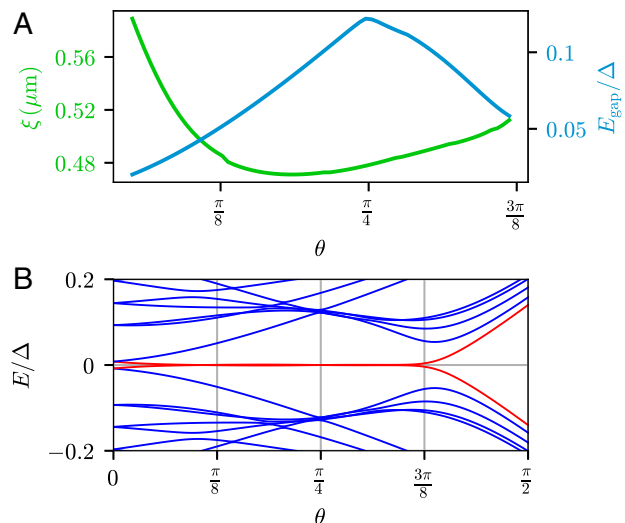
The most difficult aspect of implementing the phase-only proposal is tuning stably into the topological phase. Our parameter space consists of geometrical lengths, 2DEG density, spin-orbit strength, and superconducting phases. Geometrical lengths are set during the fabrication and can only be made uniform to within 10 nm. Thus, for a fixed geometry, we require our tun-

able parameters to bring us to a topological regime. In addition, the existence of a topological phase cannot be sensitive to small variations in lengths from unit cell to unit cell, or even within a unit cell. We have demonstrated this insensitivity in ref. 53.

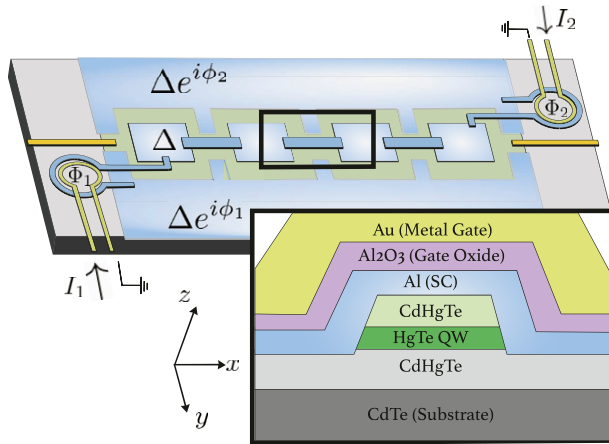
While back-gates on semiconductor quantum wells are possible (59), none of the platforms discussed above have implemented this technology while maintaining high mobility. As a result, control over the 2DEG density and spin-orbit strength are both coupled to the potential of the top gate. Gated, the mean density of HgTe can be tuned from  $4 \times 10^{11} \text{ cm}^{-2}$  to  $1 \times 10^{12} \text{ cm}^{-2}$  while the spin-orbit splitting can be tuned up to 30 meV (57, 60). In Figs. 4 and 6, the simulated density is approximately  $1 \times 10^9$  to  $6 \times 10^9$  and  $0.6 \times 10^{10} \text{ cm}^{-2}$  to  $4 \times 10^{10} \text{ cm}^{-2}$  respectively, with stable regions corresponding to approximately  $1 \times 10^9$  and  $1.4 \times 10^{10} \text{ cm}^{-2}$ . Since these densities are much less than typical semiconductor densities, and the stable regions are on the order of the fluctuations of the gate potential, we simulate more realistic conditions in SI Appendix, Fig. S3, with a density of  $5 \times 10^{11} \text{ cm}^{-2}$  showing stable regions of width  $5 \times 10^{10} \text{ cm}^{-2}$  which should be achievable with current experimental platforms.

The superconducting phases are easily tuned during operation of this device. The size of the phase-biasing loops and the current limits set the phase resolution and stability. A 10- to 20- $\mu\text{m}^2$  loop can be fabricated with ease, is robust to sub-Gauss offsets in the ambient magnetic field, and can be biased with currents of  $\leq 150 \mu\text{A}$  without causing significant Joule heating. With these parameters, a current resolution of 10 nA corresponds to a flux precision of  $2 \times 10^{-4} \phi_0$ . As shown in Fig. 4 and SI Appendix, Figs. S2 and S3, this precision is more than sufficient to tune stably into the topological regime.

As shown in Fig. 7, conductance tunnel probes can be used to detect MZMs at the ends of the junction. In many MZM tunneling experiments, the topological gap is close to the energy resolution of the tunnel probe (10  $\mu\text{eV}$ ). The large gaps in our proposal, 100  $\mu\text{eV}$  for Nb, should mitigate this issue. The type-II nature of large-gap superconductors often complicates measurements, due to small lower critical magnetic fields  $H_{c1}$  which allow



**Fig. 6.** (A) Localization length (green) and energy gap (blue) in the topological phase. The localization length is roughly inversely related to the energy gap, as expected by the continuum approximation  $\xi \approx \hbar v_F / E_{\text{gap}}$ , where  $v_F$  is the Fermi velocity. (B) Lowest-lying BdG energies as a function of the phase difference  $\theta$  in a finite system. The topological phase transition is signaled by a closing and reopening of the energy gap, while leaving two Majorana states (red) at zero energy. The parameters used are the same as in Fig. 3, with  $\phi = \pi$  and six unit cells.



**Fig. 7.** Feasible device fabrication layout. Light blue regions represent superconductors which are deposited after etching to make contact to the quantum well (QW). Superconducting connectors between center SC islands are deposited in the same step using an airbridge technique or a separate step milling through native oxides which form after the first SC deposition. The two phase degrees of freedom are set independently by current loops, which eliminates even fringing fields from the junction region which hosts the MZMs. The 2DEG is removed outside the junction region. The whole device is covered in a metal-on-oxide electrostatic gate for tuning the density/chemical potential (not shown in top view). Conductance tunnel probes, shown in orange, are included for detecting MZMs at the ends of the device.

flux penetration, motion, and trapping during device operation in external magnetic fields. However, the lack of critical magnetic fields should be an ideal use case for large-gap superconductors.<sup>‡</sup> The fact that we do not have an external field decreasing the parent gap of the superconductor should further improve the MZM visibility during tunneling experiments to the end of the junction.

## Conclusion

We have shown that MZMs can arise in a phase-controlled planar semiconductor–superconductor device without applying a Zeeman field. At the heart of our scheme lies interference between Aharonov–Casher phase (42), stemming from the spin–orbit interaction, and SC phase winding. As shown exactly by the toy model and then numerically with a realistic model, proper tuning of these two phases drives the system into a topological superconducting state.

<sup>‡</sup>Type-II superconductors also have the disadvantage for phase-based proposals of short coherence lengths. Here the relevant coherence length should be that in the semiconductor,  $\zeta_T \approx \hbar v_F / 2\pi k_B T$ , thus avoiding this problem.

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Several important advantages of our proposal are worth mentioning. It enjoys all of the benefits of planar geometry compared to nanowires, in particular, easier fabrication and the ability to generalize to a Majorana network. Moreover, since the proposal is interference based rather than Zeeman based, various limitations related to magnetic fields are lifted. In the original planar Josephson junction proposals (36, 37), the Zeeman field required to make the system topological is of the order of the Thouless energy, which scales like the inverse of the junction’s width. Therefore, making the junction narrower—which is desirable to achieve more ballistic transport—comes at the price of applying a stronger magnetic field. Here the magnetic field is not needed, making the above consideration irrelevant and facilitating the use of extremely narrow junctions. Furthermore, MZM-related experiments typically favor the use of Al due to it being a type-I SC that does not trap flux. Our proposal enables the use of larger-gap type-II SCs, such as Nb, since flux trapping is no longer a problem, thus opening the door to robust large-gap MZMs.

The discussion in the previous section shows that our proposal is realistic under existing experimental technologies. The entire class of large spin–orbit semiconductors are viable platforms, and, importantly, no topological materials are required. As we have shown, the MZM can be localized to within a single unit cell, whose size is set by the device geometry. The smallest physical dimension is set by fabrication limitations and the SOC strength (by the  $L\Delta \approx \alpha$  criterion). Thus, in the limit of large SOC, the maximal topological gap can coincide with an extremely short localization length.

In the future, it will be interesting to explore possible extensions of the design principles we suggest and investigate here in quasi-1D systems to two dimensions, for example, via a coupled-wire construction. This is expected to lead to the formation of a 2D chiral superconducting state.

**Data Availability.** All study data are included in the article and *SI Appendix*.

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